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Brief historical notes on the theory of centres of gravity

Abstract

In recent works [1] I have analyzed the historical development of the foundations of the theory of the centre of gravity during the Renaissance up until the appearance of Torricelli's (1608–1647) mathematical problem in his *Opera Geometrica* (1644). Here, I shall briefly provide you with some notes from my own studies and works in progress. I am going to explore Torricelli's organization of his mechanical theory to see if it has a remote foundation in Archimedean thought, not only for the use of such techniques as the *Reductio ad Absurdum* as seen in Archimedes' work (287–212 B.C.), but also for the logical foundations he shared with Bonaventura Cavalieri (1589–1647) seen in Torricelli's emerging analysis. In particular, I have studied the *Suppositio* and *Propositio*, set up by Archimedes from a logical and historical point of view, as rational criteria for the determination of the centres of gravity. This kind of investigation has been developed through two categories of historical interpretation: the order of ideas as an element of understanding the evolution of scientific thought on one hand; and on the other, the use of logic [2] as an element of scanning and control of the organization of the theory. This kind of examination of a theory through the use of categories is valid since the historical exploration of the foundations will not be analyzed using the traditional approach. Obviously, the content of this work could appear potentially factious, since it cannot be assumed to be the only possible perspective**.**

Key words: Archimedes's rules, *Ad Absurdum* proof, Torricelli's centre of gravity, Principle of virtual works.

(1) A short historical introduction

During the 15th century technicians and artist-engineers [3] dealt with scientific matters (*scientia activa*),[4] as well as, the theoretical science of the day (*scientia theoretica*). In addition to planning, they realized

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[1] Cf. Capecchi D. and Pisano R. (2007), "Il principio di Torricelli prima di Torricelli", *Atti del XXIV Congresso di Storia della Fisica e della Astronomia* (Napoli-Avellino), p. 107-112; *Id.* (2006a), "Torricelli e la teoria dei baricentri come fondamento della statica (submitted to *Physis*); *Id.* **(**2006b), "Reflections on Torricelli's principle in the mechanical science. Epistemology of the centre of gravity" (submitted to *Historia mathematica*).

[2] I use classical and non-classical logic as historical interpretative categories. Because the scope of this examination is very large, I cannot include all of it in this paper.

[3] Cf. Gille B. (1964), *Les ingénieurs de la Renaissance* (Paris: Hermann).

[4] My apologies to the reader if now and then I need to deal with seemingly well-known topics, already available in the broad historical and critical literature on the present subject; but a lot of the proposed thesis is still being considered in critical circles, e.g., as in some of the following texts: Dal Monte Guidobaldo (1581), *Le Meccaniche dell'Illustrissimo Sig. Guido Ubaldo dè Marchesi del Monte*, *tradotto in volgare dal Sig. Filippo Pigafetta* (Venezia). Orig Title.: Dal Monte Guidobaldo (1577), *Mechanicorum Liber* (Pesaro); Duhem P.M. (1905), *Les origines de la Statique*, Tome I (II) (Paris: Hermann), p. 91–151; Laird L.W. (1986), *The scope of renaissance mechanics*. (Philadelphia, PA USA: University of Pennsylvania Press); Dijksterhuis E.J. (1957), *Archimedes* (New York; Humanities Press); *Id.* (1961), *The mechanization of the world picture* (Oxford: Clarendon Press); Drake S. e Drabkin I.E. (1969), *Mechanics in Sixteenth Century Italy* (Madison, WI, USA: University of Wisconsin Press); Galluzzi P. (1970), *Momento. Studi galileiani*, Ateneo & Bizzarri (ed.) (Roma); Clagett M. (1959), *The Science of Mechanics in th Middle Ages* (Madison, WI, USA: University of Wisconsin Press); into Italian: *Id.* (1981), *La Scienza della meccanica nel Medioevo*, Milano: Feltrinelli), p. 43–135; Clagett M. and E. Moody (1960), *The medieval science of weights (Scentia de ponderibus* (Madison, WI, USA: University of Wiscosin Press); Clagett M., Murdoch J. (1958–59), "Medieval Mathematics, Physics, and Philosophy: A Revised Catalogue

fortifications and tools; though their knowledge was, essentially, empirical, they had ample opportunities through their work to contribute the input necessary for the search of new scientific methods in which mathematics[5] assumed more and more of an important role. Meanwhile, the study of mechanics was also being affected by the strong influence of mathematical methods. In particular, it set up firm limits on that part of mechanics, statics, which originated from the law of the lever in *Problemata mechanica*[6] (or *Quaestiones Mechanicae*). This was used in the treatment of the centres of gravity in Archimedes[7] (287–212 B.C.) and had come from the tradition of the principle of virtual works (Aristotelian and Nemorarian) and from the theory of elementary machinery.[8] The mathematical-philosophers of the 16th century began to theorize *scientia de ponderibus*, that is the science of the equilibrium of single and aggregate bodies (*aggregati*),[9] founding their ideas on essentially two theories: Archimedean statics, according to which a system of bodies is in equilibrium if its centre of gravity is bound to not fall, thus the centre of gravity is the point of a body that (under specific geometric vinculums) is able to provide it with a state of equilibrium; and Aristotelian dynamics, in which heavy bodies possess a point (*Centrum gravitates*) that naturally tends toward the centre of the World (Universe-Earth). This approach is based on the concept of *gravitas secundum situm*[10] for an "aggregato" of bodies.

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of Photographic Reproductions", *Manuscripta* (2), p. 131–154; *Manuscripta* (3), p. 19–37; Grant E. (1971), *Physical Science in the Middle Ages* (Cambridge: Cambridge Univ. Press); Weinig, P. (1996), *Science of weights in the Middle Ages: Jordanus de Nemore and his followers; Latin treatises and commentaries on balances and weights from the 13th to the 15th centuries* (Berlin: Max Planck Institute for the History of Science); very interesting is a recent article by Max Planck Institute for the History of Science regard an epistemological research of MPIWG mechanics group: Renn J., Damerow P., McLaughlin P. (2003), "Aristotle, Archimedes, Euclid, and the Origin of Mechanics: The Perspective of Historical Epistemology" (Berlin: MPIWG (ed.), pre-print 239), p. 43–59.

[5] Further on, one the first physical science affected by such influence was Renè Descartes *Optics* (1625–1637) in which any physical law was followed by a mathematical interpretation. Cf. Descartes R. (1897–1913), *Œuvres de Descartes*, par Charles Adams et Paul Tannery, Paris, 12 Voll.: see *Discours de la méthode et Essais, Specimina philosophiae* Vol. VI ; *Physico-mathematica* Vol. X, *Le Monde ou Traité de la lumière*, Vol. XI (*Id.* 1964–1974 a cura di B. Rochot, P. Costabel, J. Beaude et A. Gabbery, Paris).

[6] They may be attributed to Strato (about IV century. b.C.), one of Aristotele' pupil (about 384 – about 322). Cf. Cartelon H. (1975), "Does Aristotle A Have a Mechanics? in Barnes et al.: *Articles on Aristotle. Vol. I: Science* (London: Duckworth).

[7] Cf. Claggett M. (1964–1984), *Archimedes in the Middle Ages* (Madison – Philadelphia), *Memoirs of the American Philosophical Society*, 5 Voll. in 10 Tomes.

[8] Notoriously the general problem of the medieval mechanics was trying a solution or a reduction of the matter of the six simple machines (since *Problemata* by Pseudo-Aristotele): wheel, axle, wedge, scale, lever, inclined plane and screw. One can see: Rogers K. (2005), *On the Metaphysics of Experimental Physics*, (N.Y.: Palgrave Macmillan), p. 74–94.

[9] "Aggregato" is simile to composed parts. Particularly, (i.e.) Galileo does not explain the method of the composition. Cf. Pisano R. (2005), "*Aggregati* e *congiunti*. L'idea di un sistema di corpi in Galileo e in Torricelli", *Congr. Nazionale SIF — Italian Society of Physics*; while Torricelli will explain it in his *Opera Geometrica* (Torricelli E. (1644), *Opera geometrica* (Firenze: Massa–Landi), p. 100–102; You can also see *Id.* (1919–1944), *Opere di Evangelista Torricelli*, ed. by Loria G. e Vassura G., Voll. II, p. 103–108 (Faenza: G. Montanari); *Id.* (1975), *Opere scelte*, ed. by Bellone L. (Torino: UTET), p. 155–165.

[10] Giordano Nemorario's (about the end of XIII cent.– about 1260) conception on *gravitas secundum situm* (or *peso accidentale*), is one of the two main concepts in his *Elementa Jordani super demostrationem ponderum*; but it is possible to find them in *Liber de ratione quotedponderis* as well, by G. Nemorario (in Tartaglia edition: Tartaglia N. (1565), *Iordani Opusculum de ponderositate Nicolai Tartaleae studio correctum, nouisque figuris auctum,* apud Curtium Troianum, Venetia). The second one is about the relationship between the movements of the application points of the weight-forces and their value. For a more in depth examination *gravitas secundum situm*, Principle of virtual works and *Scientia de ponderibus* about Jordanus: Cf. Duhem P.M. (1905), Vol. II; Ginzberg B. (1936), "Duhem and Jordanus Nemorarius," *Isis*, vol. 25, p. 341–362; Clagett M. (1961), *op. cit*; Thomson R. B. (1976), "Jordanus de Nemore: Opera", *Mediaeval* Studies, vol. 36, p. 97–144; Clagett M. and Murdoch J. (1958–59), p. 131–154; p. 19–37, *op. cit.*; Brown J. E. (1967–68), *The "Scientia de ponderibus" in the later middle ages* (Ann Arbor: UMI PRESS), Vol. 2; Grant E. and Murdoch J. E. (1987), *Mathematics and its applications to science and natural philosophy in the Middle Ages: essays in honor of Marshall Clagett* (Cambridge: Cambridge University Press).

According to this idea, a group of bodies is in equilibrium if (under certain conditions) the tendencies of the various bodies can balance themselves out (i.e., through the equality of virtual works):[11]

> If two weights descend along diversely inclined planes, then, if the inclinations are directly proportional to the weights, [then] they will be of equal force in descending [that is they will apply the same force, from which we have equilibrium].[12]

Archimedes' (287–212 B.C.) contribution is really fundamental for a historical study of the history of the foundations of the centre of gravity and Torricelli's principle. The great man from Syracuse was the first scientist to set up *rational criteria* for determining centres of gravity. For my purposes, I have mainly examined, *Book I* [13] in *On Plane Equilibrium* where Archimedes [14] in addition to studying the rules of the Law of the Lever, he also defines the centre of gravity for the parallelogram, the triangle and the trapezium as well. He gives us the basic elements of the theory of the centre of gravity by establishing seven hypotheses for the theory.[15] Using these hypotheses, Archimedes is able to define the *rational criteria* or *Propositions* to "calculate" the centre of gravity of composed bodies, starting from the knowledge of the centre of gravity of the single bodies from which they are formed:

[*Suppositio*] We posit

1. Equal weights suspended at equal distances [from fulcrum] are in equilibrium; equal weights suspended at unequal distances [from fulcrum] are not in equilibrium but [they] incline towards the weight suspended at the greater distance [from fulcrum].
2. When weights are in equilibrium at certain distances [from fulcrum], if something is added to one of them they will not be [yet] in equilibrium but will incline toward the weight to which something is added.
3. Similarly, if something is taken away from one of the weights, they are not in equilibrium but incline toward the weight from which nothing was taken.
4. In two equal, similar, and coinciding plane figures the centers of gravity also mutually coincide.

[11] Cf. Duhem P. (1905–06), T. II, p. 1–99, *op. cit.*; Clagett M. (1959), p. 123–124, *op. cit.*; 179–183; Mach E. (2001), *La meccanica nel suo sviluppo storico-critico*, Bollati Boringhieri (ed.), (Torino), p. 81–101. E.02 Quod gravius est, velocius descendere (that which is heavier descends more rapidly); E.03 Gravius esse in descendendo, quando eiusdem motus ad medium rectior est. (It is heavier in descending, when its motion toward the centre is more direct); E.04 Secundum situm gravius esse, quando in eodem situ minus obliquus est descensus. (It is heavier for position, when, at a given position, its path of descent is less oblique); E.05 Obliquiorem autem descensum, in eadem quantitate minus capere de directo. (A more oblique descent is one in which, for a given distance, there is a smaller component of the vertical). (Clagett M., E. Moody (1960), p. 128–130, 379, *op. cit.* Notes in brackets are by Clagett M. (1981), 94, line 30, *op. cit.*).

[12] "Quaestio decima. Si per diversarum obliquitatum vias duo pondera descendant, fueritque declinationum et ponderum una proportio eodem ordine sumpta, una eritutriusque in discendendo. (*Liber de ratione ponderis* edited by Tartaglia N. 1565, *Iordani Opvscvlvm de Ponderositate*, Nicolai Tartaleae Stvdio Correctvm Novisqve Figvrisavctvm Cvm Privilegio Traiano Cvrtio, Venetiis, Apvd Curtivm Troianvm. M D Lxv, 7, line 2). I found it more exhaustive reporting the concept of *gravitas secundum situm*. You can also see *Elementa Jordani in* Clagett M., Moody E. A. (1952), p. 191, *op. cit.*

[13] In *Book II*, Archimedes deals with the centre of gravity of the parabola segments.

[14] Cf. Clagett M. (1959), p. 30–31, *op. cit.;* Cf. *Id.*, *Archimedes in the Middle Ages* (1964), Vol. I, p. 116–120, *op. cit.*; Cf. Id., (1981), p. 50–56; Dijkstrehuis E.J. (1956), p. 21–33, p. 286–359, *op. cit.* The *Method* is the work of Archimedes who more than anyone defines the basis "for the way to the discovery". Cf. Frajese A. (1969), *Attraverso la storia della matematica* (Le Monnier), p. 261–296; see also *Id.* (1964), *Galileo matematico*, (Roma: Editrice Studium), capp. IX, X, XII; Heath T. L. (2002), *The works of Archimedes* (N.Y: Dover Publications INC, Mineola), p. 189–219; Favaro A. (1923), *Archimede. — OPERE*, A. F. Formiggini, (ed.) (Roma).

[15] The centre of gravity in Archimedes was referred to bodies which were operatively composed until they became only one, which was given by sum of all the others and for which it was attempted to define the total centre of gravity. In particular the sum of all the components may require the adoption of the method of exhaustion.

5. The center of gravity of unequal but similar weights will be similarly situated. We say that points are similarly situated in relation to similar figures when the straight lines drawn from these points to equal angles make equal angles with the corresponding sides.
6. If magnitudes are in equilibrium when suspended at certain distances, then magnitudes equal to them, suspended at the same distances [from fulcrum], will also be in equilibrium.
7. The center of gravity of every figure whose perimeter is concave in the same direction is necessarily [placed] inside of the [same] figure.

With these [previous seven] *Suppositio*, [we propose the following seven *Propositio*:[16]]

1. Weights suspended from equal distance [from fulcrum are] in equilibrium, [then they are] are equal.
2. Unequal weights suspended at equal distances [from fulcrum] are not in equilibrium but [they] incline toward the greater weight [of them].
3. Unequal weights suspended at unequal distances [from fulcrum] can be in equilibrium, [and] the greater weight [of them] being suspended at the lesser distance [from fulcrum].
4. If two equal magnitudes do not have the same center of gravity, the center of gravity of the magnitude composed of both these magnitudes is at the middle point of the straight line joining the centers of gravity of the magnitudes.
5. If the centers of gravity of three magnitudes are situated on the same straight line, and the magnitudes are equal in weight, and the distances between the centres [of gravity of three magnitudes] are equal, [the] center of gravity of the composite of all the [three] magnitudes will be a point which is also the center of gravity of the middle [magnitude].
6. [Two] commensurable magnitudes are in equilibrium when they are inversely proportional to the distances at which they are suspended.
7. But also, if the magnitudes are incommensurable, they will similarly be in equilibrium when suspended at distances inversely proportional to their magnitudes.

It should be noted that Archimedes considers *Supposition I* as already mentioned ("the weights are in equilibrium"), as the sufficient condition for equilibrium and not *Proposition I* ("the weights are equal"), which provides the necessary condition for equilibrium.[17] Indeed the doubt can exist that equilibrium may also exist in the case of different weights. He assumed as more evident sufficient conditions for equilibrium with respect to necessary ones; this will be the same position assumed by Torricelli. It is worth remarking that in both either the static or dynamic approaches, the existence and uniqueness of the centre of gravity is involved. Nevertheless, the static view seems better than the dynamic one, since it is supported by rational criteria for determining the centre of gravity of an "aggregato" of bodies, once you realize the centre of gravity of every single body. The theory providing such criteria, barystatics (or centre of gravity), is largely based upon the results of the Theory of the Lever developed by Archimedes.[18] Archimedes built a theory based on a specific problem: the study of the centre of gravity of a body, or two or more composed bodies, and the attempt to determine the centre of gravity by application of rational criteria. This problematical approach seems to project Archimedes away from the Euclidian-Aristotelian (axiomatic) model, made mainly of deductive principles that are self-

[16] Clagett M. (1959), *op. cit.* I studied and comment seven *Propositions by* adapting Clagett's translation and commentary. Cfr. *Id.*, p. 32–37; *Id.* and Moody E. (1960), *op. cit.*

[17] Probably Archimedes considered it *studied* for geometric construction or just previous mentioned; maybe one can think that he already dealt with it in the lost treatise *on the Lever*. Cf. Heath T.L. (2002), p. 191, *op. cit.* Surely, he (in *On Plane Equilibrium*), mostly write about *criteria to calculate a centre of gravity*, and not with *definitions* (i.e.) what is the centre of gravity of a body.

[18] Cf. Clagett M. (1964), Vol. I, p. 116–120, *op. cit*; Dijkstrehuis E.J. (1956), *Archimedes* (Copenaghen: Ejnar Munksgaard), p. 21–33, 286–359. In *Book II*, Archimedes deals with the centre of gravity of the parabola segments. The opera *Method* is the work of Archimedes who more than anyone defines the basis "for the way to the discovery" (Cf. Frajese A. (1969), *Attraverso la storia della matematica* (Firenze: Le Monnier (ed.), 261–296*;* see also *Id.* (1964), *Galileo matematico* (Roma: Editrice Studium), capp. IX, X, XII).

evident to reasoning. He instead tried to base his theory on mechanical and empirical affirmations that did not have the evident characteristics of axiomatic Euclidean principles.

(2) What is an Archimedean paradigm?

Since the 13th century, the use of virtual motion and of virtual works in the treatment of statics were of notable importance, not only for the generalization of the results attainable by adopting such principles, but also because they was generally associated with the use of *ad absurdum* reasoning. Such reasoning establishes a particular way of conceiving and managing the structured order of a scientific theory. This problematical approach seems to move Archimedean thought toward a new paradigm of science unlike the Euclidian-Aristotelian model. Archimedes built his theories upon mechanical and empirical affirmations that did not contain characteristic axiomatic Euclidean principles. From another point of view, some authors define this as a new "mental model:"

> According to our analysis of Aristotle's text [*Mechanical problems*] the knowledge structures it displays emerged from a reflection of experiences made possible by the invention of the balance with unequal arms, an invention that had taken place only recently.[19] These knowledge structures are determined by a specific mental model resulting from an integration of mechanae model[20] with the equilibrium model, a model that we have called "the balance-lever model". This model can indeed be understood as a generalization of the equilibrium model [Aristotelian school model] associated with the ordinary balance with equal arms. In the case of an equal arms-balance, weight difference are balanced by weights; in the case of an unequal arms-balance, they are balanced by changing the position of the counterweight along the scale or, as in Aristotle's case, by fixing the counterweight at the end of the beam and changing the position of the sus-pension point. This necessarily generalized the equilibrium model: weights can be compensated not only by weights but also by distances. […].[21] [When it concern to] Archimedes' concept of magnitude, in connection with the concept of weight and centre of gravity, indeed works like the mental model […], we shall refer to the corresponding model as the "centre of gravity model". It can be applied to any heavy body, allowing us mentally to replace it by its total weight and its centre of gravity. It slots are therefore the heavy body itself, its total weight, and the centre of gravity. The structure of the model is determined by noting that any axis through the centre of gravity turns the body into a lever in equilibrium […]. In other words, the [Archimedean] centre of gravity model [Problematical theory] allows any body to be conceived as a generalized balance with fulcrum and distribution of weighs around it in equilibrium. In contrast to the fulcrum [Aristotelian theory], however, the centre of gravity no longer has to be a physically distinguished point that can be identified by visual cues but its identification is rather the result of the application of the model to a heavy body.[22]

Simeon Stevin [23] (1548–1620) was the first one to accept, though with some doubts, the validity of Archimedes' theory. He also developed the theory of the centres of gravity further. In particular, he widened the question of the equilibrium of a body bound to vertical motions or to any type of action. It is not immediately clear whether he did or did not use virtual motions; he might have even definitively rejected them.[24] The opposite view is provided by Ernst Mach (1838–1916), in his famous text on

[19] Cf. Renn J., Damerow P., Rieger S. (2002), "Mechanical Knowledge and Pompeian Balances" in *Homo Faber: Studies on Nature, Technology, and Science at the Time of Pompeii,* Castagnetti G. and Renn J. (eds.), p. 3–18 (Rome: L'Erma di Bretschneider).

[20] Cf. Renn J., Damerow P., McLaughlin P. (2003), p. 46, line 26, *op. cit*.

[21] *Id.*, p. 47, line 10.

[22] *Id.*, p. 51, line 12.

[23] According to Ernst Mach (1838–1916), Simeon Stevin (1548–1620) in his *Mathematicorum hypomnematum de statica* (Stevin S. (1605), *op. cit.*) was the first one to consider the problem of the equilibrium from a mechanical point of the view, applying it to *Archimedes' pulley* (cf. Mach E. (2001), p. 80, *op. cit.*).

[24] Cf. Dijksterhuis E.J. (1980), p. 434, *op. cit.*

mechanics [25] where he presumes that "the spring of the principle of virtual motions" was involved in Stevin's *Tomus Quartus Mathematicorum Hypomnematum de Statica* (1605):

> In a [geometrical] system of pulleys is in *equilibrium*, the products of each weight and sizes of their respective displacements are equals .[26]

According to Marshall Clagett, Simeon Stevin tried to exclude the dynamic principle from the science of statics in which one cannot find a motion by definition, reserving "as Erone, […] the principle of work or its equivalent to the explanation of the mechanical advantage offered by machines."[27] The two approaches, static and dynamic, were sided with alternate events. At the beginning, the Aristotelian approach was essentially supported by mathematician-philosophers, also because of the theoretical weakness of the counterpoising approach of engineers-artisans. It seems that Simeon Stevin, who himself rejected the dynamic approach for determining the equilibrium of the bodies, would use it in his famous demonstration of the inclined plane, based upon the impossibility of a perpetual motion.[28] Galileo Galilei (1564–1642) used both methods as well; [29] and, although he was a fervent supporter of the Archimedean way, he was not exempt from Aristotelian influences when in the *Mechanichs* he demonstrates the Law of the Lever using a dynamic approach.[30] He also dealt with virtual motions on several other occasions as well.[31]

Viewed thus, Archimedes' theory seem to belongs to problematic theory compared to the Aristotelian and Euclidean approach which is organized along the lines of geometrical and dynamical theories that show their universal view of the world. In fact, as has just been mentioned, Archimedes founded his theory upon an empirical choice of focusing on a problem to find the centre of gravity of two or more magnitudes and to find the solution by means of inductive reasoning. This view is well confirmed in his *Method*, and in particular, to understand the behaviour of a lever when the weights are forces.

Finally, in 1644, Evangelista Torricelli (1608–1647) in his *Opera Geometrica* states a rational criterion which played a critical role in the history of mechanics and can be viewed with no doubt as the springboard for the modern principle of virtual works: [32]

> *Praemittimus* Two "coniucta" bodies cannot move by themselves, their common centre of gravity does not descend.[33]

Strangely enough, Torricelli did not seem to realize the basic relevance of his *Praemittimus*. In fact, it arose from the necessity of proving a theorem about the inclined plane, which, in his opinion, Galileo Galilei had neglected to clear up in the dynamic of parabolic motions:[34]

[25] Mach E. (2001), p. 80, line 14, *op. cit.*

[26] "Ut Spatium agentis, ad spatium patientis: sic potentia patientis, ad potentiam agentis" ("Additamenti Statica Pars Secvnda: De Trochleostatica", in Stevin S., *Hypomnemata Mathematica* ... (1605–08), L. 4, p. 172, line 3, *op. cit.*.

[27] Clagett M. (1981), p. 37, *op. cit.*

[28] "[…] ipsique globi ex sese continuum et aeternum motion efficient, quod est falsum". (Stevin S. (1605–1608), "Liber primus Staticae. De staticae elmentis", in: *Tomus quartus mathematicorum hypomnematum de statica* (Lugodini Batavorum), line 10, 35). See also Dijksterhuis E.J. (1955), *The principal works of Simon Stevin*, vol. I, "Mechanics", N.V. Swets & Zeitlinger, (ed.), p. 179, in: Clagett M. (1981), p. 123, n. 54, *op. cit.*).

[29] Galilei G. (1890–1909), *Opere di Galileo Galilei*, ed. Naz. by A. Favaro, 20 Voll; *Id.*, Vol. VII, p. 146–188; *Id.*, Vol. IV, p. 63–140, *op. cit.*

[30] *Id.*, Vol. II, p. 163–186, *op. cit.*

[31] *Id.*, Vol. II, p. 240–242; *Id.*, Vol. IV, p. 68–69; *Id.*, Vol. VIII, 310–331, 329–330, *op. cit.*

[32] Cf. Capecchi D. (2000), *Storia del principio dei lavori virtuali da Arsistotele a Bernoulli* (Napoli: Luda).

[33] "Praemittimus. Duo gravia simul coniuncta ex se moveri non posse, nisi centrum commune gravitatis ipsorum descendat". (Torricelli E. (1644), *Libro II*, 99, line 4, *op. cit.*). In other words, a system of bodies is in *equilibrium* if its common centre of gravity cannot sink in any of its possible motions.

[34] "[…] Definitiones omisimus et genere scriptionis contracto, laconicoqueusi sumus, quia dum universam Galilei doctrinam pro suppositione praemittimus lectori erudito profitemur" Acturus de Motu naturalitèr accelerato Galileus principium supponit, quod et ipse non admodum evidens putat, dum illud parum exacte penduli experimento nititur comprobare, hoc est: *Gradus velocitatis eiusdem mobilis super diversas planorum inlinationes*

> The speeds acquired by one and the same body moving down planes of different inclinations are equal when the heights of these planes are equal.[35]

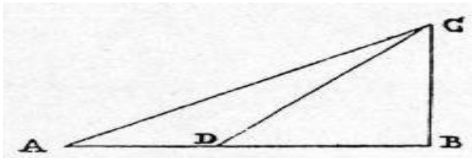

Fig.1 Galilei's theorem[36]

According to his *Praemittimus*, Torricelli proves his theorem using the following preposition:

> *Propositio II*. The momentum of the same bodies along [two] unequal planes [but] having equal inclination, are in reciprocal proportion to the lengths of the [two planes].[37]

Torricelli thought that the descent of a body of mass *m* along the cathetus BC, i.e., equal to a quantity *S*, was sufficient for a body of a mass *M* to move up along the hypotenuse[38] *AB* for $S\sin\alpha$ quantity (Fig. 1):

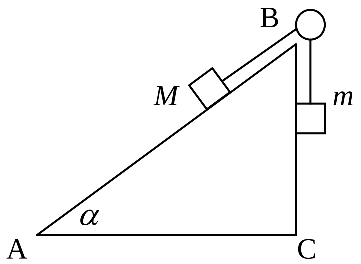

Fig. 2. *Equilibrium* on inclined plane

The condition of equilibrium[39] is:

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acquisitos, tunc esse aequales cum eorumdem eorwrndern planorum elevationes aequàles sint. Ex hac petitione dependet quasi universa illius doctrina de motu tùmi accelerato, tum proiectorum. Si quis de principio [Galilei's] dubitet de ijs quae inde consequuntur certam omninò scientiam non habebit. ([Torricelli quotes Galilei's theorem from Mechanics] Momenta gravium aequalium super planis inequaliter inclinatis, esse inter se ut sunt perpendicula partium aequalium eorumdem planorum. [From which, according to Torricelli, one can tried to demonstrate, that is] Nos quia in huiusmodi Theorema non incidimus, hoc primum aliqua demonstratione confirmabimus: protinus add ostedendum id quod Galileo principium sive petitio est accedemus. (Torricelli E. (1644), 98, line 9, *op. cit.*; italic style by Torricelli). See also:

"SALV. Fermata cotal definizione, un solo principio domanda e suppone per vero l'autore, cioè: Accipio Gradus velocitatis eiusdem mobilis super diversas planorum inclinationes acquisitos tunc esse aequales, cum eorumdem planorum elevationes aequales sint" (Galilei G. (1990), *Discorsi e dimostrazioni matematiche*, G. III, E. Giusti editor, p. 179, l. 25, *op. cit.*).

Let me note that Galilei re-formulated it by means of Principle of virtual works. (Cf. Caverni R. *Storia del metodo sperimentale in Itali*a, vol. IV, 239–241, *op. cit.*).

[35] "Accipio, gradus velocitatis eiusdem mobilis super diversas planorum inclinationes acquisitos tunc esse aequales, cum eorumdem planorum elevationes aequales sint". (Galilei G. (1890–1909), *Opere*, Edizione Nazionale by A. Favaro (Firenze: G. Barbèra), Vol. VIII, G. III, 205, line 9).

[36] *Ibidem*.

[37] "Propositio II. Momenta gravium aequalium super planis inaequaliter inclinatis, eandem tamen elevationem habentibus, sunt in reciproca ratione cum longitudinibus planorum". (Torricelli E., *Libro II* (1644), p. 100, line 21, *op. cit.*).

[38] You can imply consider that to the point *B* to binding device is necessary that it allows the slide of the rope. According to the original treatment, I have neglected the friction so geometrical aspect is more evident.

[39] Cf. Mach E. (2001), p. 79–83, *op. cit.*

$$mS - MS\sin\alpha = 0$$

$$m = M\sin\alpha = M\,\frac{BC}{AB}\cdot$$

If the ratio of the weights changes, then the centre of gravity will have the tendency to move and the equilibrium will stop. Nevertheless, despite the breadth and importance of the scientific literature he has left us, and notwithstanding his premature death, Torricelli has attracted little interest among 20th century historians and most studies focus primarily on commemorative and mathematical themes [40] regarding mathematical matters: O*n the spiral*, *On the methods of the tangent* and *On the matter of indivisibles*, just to cite a few. Among them, a work by A. Agostini [41] called (in English) *Centres of Gravity Found by Torricelli*, seems particularly interesting. It is a historical review of parts of some of Torricelli's letters sent to prove the results of his studies about his principle in mechanics.

What is interesting here is the attempt to understand how Torricelli's theory of the centres of gravity was influenced by Archimedes' work. We are left with a question about whether or not Torricelli's approach is simply the result of the acquisition of mathematical techniques similar to Archimedes or if it is also possibly a new way of conceiving science in terms of the organization of the theory? But this is a work in progress …

[40] Cf. Agostini A., (1950), "Il metodo delle tangenti fondato sopra la dottrina dei moti nelle opere di Torricelli", *Period. Mat.* (4) 28, p. 141–158; *Id.*, (1951a), "I baricentri trovati da Torricelli", *U.M.I Journal*, p. 149–159; *Id.*, (1951b), "Il "De tactionibus" di Evangelista Torricelli", *Boll. Un. Mat. Ital*., (3) 319–321; *Id.*, (1951c), "Problemi di massimo e minimo nella corrispondenza di E Torricelli", *Rivista Mat. Univ. Parma*, (2), p. 265–275; Blay M. (1985), "Varignon et la status de la loi de Torricelli", *Arch. Int., Hist., Sc.*, Vol. 35, p. 330–345; Bortolotti E. (1922), *Applicazioni del calcolo integrale alla determinazione del centro di gravità di figure geometriche*, Accademia di Bologna; *Id.* (1943), "Il problema della tangente nell'opera geometrica di Evangelista Torricelli", *Mem. Accad. Sci. Ist. Bologna. Cl. Sci. Fis.* (9), 10, p. 181–191; Baroncelli G. (1993), "On the invention of the geometric spiral: an unpublished letter of Torricelli to Michelangelo Ricci", *Nuncius Ann. Storia Sci.*, 8 (2), p. 601–606; De Gandt F. (1992), "L'évolution de la théorie des indivisibles et l'apport de Torricelli", *Geometry and atomism in the Galilean school* (Florence), p. 103–118; Medolla G. (1993), "Alcuni documenti inediti relativi alla vita di Evangelista Torricelli", *Boll. St. Sc. Mat.*, Vol. XIII (2), p. 287–295; Krarup J. Vajda S. (1997), "On Torricelli's geometrical solution to a problem of Fermat, Duality in practice", *IMA J. Math. Appl. Bus. Indust.* 8, (3), p. 215–224; Segre M. (1983), "Torricelli's correspondence on ballistics", *Ann. of Science* 40 (5), p. 489–499; Tenca L. (1958), "L'attività matematica di Evangelista Torricelli", *Periodico di Matematiche,* (4), Vol. 36, p. 251–263.

[41] Agostini A. (1951a), p. 149–159, *op. cit.*