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Geometrical studies in 17th century Spain and their counterparts in European mathematics

MATHEMATICS IN SPAIN DURING the 16th and 17th centuries are mainly of the applied and instrumental kind, they are used in astronomy, military fortification, cartography, navigation, engineering, trade, and so on, and therefore most of the books written were intended for teaching basic arithmetic, Euclid's *Elements*, commercial calculus, the first rules of algebra, trigonometry, and logarithms. Spain did not participate in the progress that algebra and calculus were making in Europe throughout the 17th century, but there were studies and scientific work in the field of classical geometry.[1] The written works were few indeed and they had little diffusion. However, attention should be paid to these few works because, on the one hand, they themselves have an intrinsic value and, on the other hand, they are part of our mathematical heritage. The existence of all these works is a fact, whether they influenced much, little or in no way the mathematical trend that nowadays is regarded as the main one.

The works on classical geometry in 17th century Spain were all but for one the result of the research led by professors of the Imperial College in Madrid.[2] Some of these works were not published, but are kept in manuscript version and other works are lost but referred to by other authors. In chronological order, the most important geometers of the period were Claude Richard, Jean Charles de La Faille, José Zaragozà, Hugo de Omerique and Jacobo Kresa.[3] All of them were Jesuit except for De Omerique who, nevertheless, was educated in the Jesuit College in Cádiz. They all worked geometry by means of the Eudoxian theory of proportions. Rarely did they make use of algebra and, when they did, they did not follow Descartes. However, not using algebra in Descartes' way is no reason for us to neglect these works, even more when it was throughout the 17th century that Europe was discovering algebra as a good operative language for mathematics. The topics of pure geometry covered by the geometers of the Imperial College were also dealt with by other European mathematicians such as Fermat, Huygens, Tacquet, Viviani, Borelli, Ceva and many others.

At the end of the 16th century, European mathematics recovered the Greek past through direct translations from Greek into Latin, and there was a special interest in discovering and understanding the works of Apollonius. Federigo Commandino published in 1588 in Pisa *Alexandrini Mathematicae Collectiones*, which was the translation into Latin of Pappus of Alexandria's *Synagoge* (c. 300 B.C.). The *Synagoge* was written with the aim of reviving Greek Classical geometry. The book included geometrical propositions by authors such as Archimedes, Euclid, Apollonius, Hippias, Autolycus, Aristarchus and many others. Most results were just enunciations and there appeared no demonstration. If one was interested in the demonstrations, they could be found in the original books which at the time were kept in the library of Alexandria. In the 16th century, many of those originals had been lost and hence the importance of Pappus' book. Thanks to it, many 17th c. mathematicians had news about what was done in high level Greek Classical Geometry. In some cases, Pappus used to include together

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[1] There are two works that are not geometrical: *Pharus Scientiarum* (1659) by Sebastian Izquierdo, and *Kybeia*, which is a part of *Mathesis biceps* by Caramuel. The former deals with combinatory, and the latter is a treaty of 22 pages on calculus of probabilities that can be considered as the second treaty on this subject after Huygens' work. See Navarro Brotons (1996).

[2] The Imperial College in Madrid was an educational institution directed by the Society of Jesus and its main goal was the education of the nobility. See Navarro Brotons (1996).

[3] Claudio Richard (b. Ornans 1589, Franche Comté, d. Madrid 1664); Jean Charles de La Faille (b. Antwerp, 1597, d. Barcelona 1652); José Zaragozà (b. Alcalà de Xivert, Castellón 1627, d Madrid 1679); Hugo de Omerique (b. Sanlúcar de Barrameda, Cádiz 1634); Jacobo Kresa (b. Smirschitz, Austria 1645, d. Brunn, Austria 1715).

with the enunciations of the propositions previous lemmas which he demonstrated and which were helpful to understand the texts. Interestingly enough, what in principle was a lack for 17th c. mathematicians — the lack of the original texts — turned out to be a great motivation that inspired many of 17th c. works on geometry, which tried to recover the Ancient wisdom. On the one hand, they reconstructed conjecturally lost works; and on the other hand, new methods were introduced in order to demonstrate all those propositions of which only the enunciation remained. Vieta, Ghetaldi, Snell, Fermat, Descartes, Pascal, Schooten and many others took part in this task. The development of mathematics in 17th century Spain resulted also from the interest in recovering the Greek heritage. In this paper I am bringing to light some geometrical works, authors and most noteworthy results of 17th century Spain, and I shall relate them to other similar works that were being developed outside Spain.

The most outstanding works in geometry in 17th c. Spain are the following:

Jean Charles de La Faille's
Theorema de centro gravitatis partium circuli et ellipsis (Antwerp 1632)
Tratado de las secciones cónicas (non-published manuscript)

Claudio Richard's
Euclidis Elementorum geometricorum (Antwerp 1645)
Apollonii Pergaei Conicorum libri IV, cum commentariis R.P. Claudii Richard (Antwerp 1655)

José Zaragozà's
Trigonometria española, resolución de los triángulos planos, y esféricos (Mallorca 1672)
Eucides Novo-Antiquus Singulari Método Illustratus (Valencia 1673)
Geometria Magna in Minimis (Toledo 1674)
Euclides Nuevo-Antiguo,Geometria Especulativa y Práctica de los Planos y Sólidos (Madrid 1678)
Loca Plana Apollonii Pergaea (non-published manuscript)
De ellipsi et circulo (non-published manuscript)

Jacobo Kresa's
Elementos Geométricos de Euclides, los seisprimeros libros de los planos y los onzeno y dozeno de los sólidos.Con algunos selectos teoremas de Arquímedes (Brussels 1689)

Hugo de Omerique's
Analysis Geometrica sive nova, et vera methodus resolvendi tam problemate Geometrica, quam arithmeticas quaestiones. Pars Prima: De planis (Cádiz 1698)

These works can be placed together according to the following three thematic groups:

1. Didactic versions of some geometrical subjects, some of which include new results or new perspectives.
   Tratado de las secciones cónicas (La Faille)
   Euclides Novo-Antiquus Singulari Método Illustratus (Zaragozà)
   Euclides Nuevo-Antiguo, Geometria Especulativa y Práctica de los Planos y Sólidos (Zaragozà)
   Trigonometria española, resolución de los triángulos planos, y esféricos. Mallorca 1672 (Zaragozà)
   De ellipsi et circulo (Zaragozà)
   Geometria Magna in Minimis (Zaragozà)
   Elementos Geométricos de Euclides, los seisprimeros libros de los planos y los onzeno y dozeno de los sólidos.Con algunos selectos teoremas de Arquímedes (Kresa)
2. Annotated versions or restitutions of classical works.
   Euclidis Elementorum geometricorum (Richard)
   Apollonii Pergaei Conicorum libri IV (Richard)
   Geometria Magna in Minimis (Zaragozà)
   Loca Plana Apollonii Pergaea (Zaragozà)
   Analysis Geometrica (Omerique)
3. Works that contain new methods and new results as regards pure geometry.
   Theorema de centro gravitatis partium circuli et ellipsis (La Faille)
   Geometria Magna in Minimis (Zaragozà)
   Analysis Geometrica (Omerique)

I shall limit myself now to comment on four of the cited works and I shall begin with *Geometria Magna in Minimis* (GMm from now on), which is the less known, although it contains the biggest number of new results in pure geometry. If GMm was just read by a few and had no diffusion is probably due to the fact that it develops a theory that was too abstract and lacking of applicability.[4] Some of the results of GMm were later discovered by non-Spanish mathematicians and the paternity of such results is still attributed to them.

GMm consists of three volumes in which Zaragozà develops the construction of a method based on properties of the barycentric kind. In order to establish the new method, Zaragozà introduced a concept that is the most original of his work, namely, the concept of Centrum Minimum of points and polygons:

> Centrum minimum, dicitur punctum, ex quo prodeunt recta ad qodlibet data puncta, utcumque disposita, supra quas figurae constitutae, licet interse dissimiles datis tamen similes, minimam omnium similium summam efficiunt (Chapter 2, GMm I).[5]

Zaragozà's aim was to define a point whose properties are the same as those of the centre of gravity, but without using physical properties. What is more, he wanted to define this point in Euclidean terms. Such a point is not found in Euclid's *Elements* nor is it found in any book written before the 19th century. Zaragozà introduced the centrum minimum in order to solve a geometrical locus ("locus II-5" from now on) given by Apollonius in *Loci Plani*. This locus was known by the mathematicians of the 17th century through the Latin translation by Comandino of Pappus' *Synagoge*, where the named locus was only given, but not demonstrated. The locus II-5 was enunciated as follows:

> Si a quotcumque datis punctis ad punctum unum inflectantur rectae lineae et sint species, quae ab omnibus fiunt data spatio aequales punctum continget positione datam circumferentiam.[6]

Different mathematicians of the 17th century were interested in its demonstration. Around 1636, the locus II-5 was dealt with by the French mathematician Fermat in his restitution of *Loci Plani*.[7] In 1656, the Dutch mathematician Schooten, author of the Latin version of Descartes' *Geometrie*, solved the locus II-5 as an applied example of Descartes' analytical method.[8] Also in Holland, Huygens called Apollonius' locus 'propositio mirabilis' and applied it to the study of the isochrony of flat figures. His solution of the locus II-5 is also analytical and appears in *Horologium Oscillatorium* (1673).

Notice that Fermat, Schooten as well as Huygens slightly simplified Apollonius' locus in their resolution.[9] Therefore, when in the 18th century the Scot geometer Simson published one restitution of the *Loci Plani*,[10] in the preface he stated that it was him who for the first time had recovered and demonstrated Apollonius' locus II-5 in its original terms, in contrast with Fermat's, Simson's and Huygens' solutions. However, Simson made no reference to the restitution which Zaragozà had already made seventy five years earlier.[11] The 19th century French mathematician M. Chasles, in his book of 1837, *Aperçu Historique sur l'origine et le developpement des métodes en géometrie*, gives Simson's restitution as a model of restitution of Apollonius' locus II-5, but he makes no reference to Zaragozà.

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4 Something similar happened in France with Desargues' *Brouillon projet*.

5 Centrum minimum is that point from which if straight lines are drawn to the given points, the sum of the figures (given in species) described on all of them is a minimum.

6 If from any number of given points whatever straight lines be inflected to one point and the figures given in species described on all of them be together equal to a given area, the point will be on a circumference given in position. (Heath's translation, 1981, p. 188).

7 The restitution by Fermat appears in his *Complete Works*, which were published in 1679.

8 Schooten's resolution is found in his *Exercitationum Mathematicarum*.

9 Their simplified version of Apollonius' locus II-5 says: If from the centre of gravity of a number of arbitrary coplanar points a circumference is drawn, then the sum of the squares of the rectilinear segments that go from any arbitrary point on the circumference to the given points is always the same.

10 Simson's restitution is found in his *Apollonii Pergaei Locorum Planorum libri II* (Glasgow, 1749).

11 Although Simson was not aware of Zaragozà's resolutions, it is not surprising that his and Zaragozà's resolutions were similar, for they both started from Pappus' *Synagoge*.

A noteworthy result of GMm is found in the proposition 37 of the second volume. In this proposition Zaragozà shows what is nowadays known as Ceva's Theorem, although Ceva's result was published four years later.[12] While Ceva arrives to this result using the physical properties of the centre of gravity, Zaragozà uses the geometrical properties of the centrum minimum, but the ways in which they both proceed are very similar as regards the use of barycentric properties. Both Ceva and Zaragozà were neglected in their time, but, whereas the Italian Ceva was recovered and positively valued by the French M. Chasles[13] in the 19th century, Zaragozà's work was still neglected. This is why nowadays the result to which they both arrived is known with Ceva's name. Before Ceva, the result had been attributed to John Bernouilli.[14]

In the 19th century, in Germany, Möbius wrote *Der barycentrische Calcul* (Leipzig 1827). In this book, making use of algebra, he further developed the barycentric method that Zaragozà had introduced. But Möbius never mentions Zaragozà.

Throughout GMm, Zaragozà uses the barycentric method to deal with many other topics of classical geometry. I shall give only two examples. Firstly, he studies the partition of polygons in proportional parts, following the Belgian Jesuit Andrea Tacquet in his *Geometria Practica*. However, Zaragozà went beyond and studies the partition of some three-dimensional figures. Secondly, Zaragozá calculates quadratic relations between sides and diagonals of polygons. It is at this point that he demonstrated a result[15] that Euler demonstrated in the following century. Once more this result is not attributed to Zaragozà.[16]

Let us now move to the second geometrical work worth mentioning: *Analysis Geometrica* by Hugo de Omerique. In this work, Omerique follows Vieta, Ghetaldi and others as regards their attempts at recovering Greek geometry. Inspired by the method of analysis of the ancient Greeks described by Pappus in book VII of the *Synagoge*, Omerique established a model of this method. He uses rectilinear segments and Eudoxian proportions to show propositions and to solve problems. As it is well known, Descartes also recovered the method of analysis of the ancient Greeks, but he changed it considerably, for — after taking a segment as the unit — he could define a product of segments in terms of another segment. This led him to incorporate algebra as an operative language to manipulate geometry. Despite the revolutionary step of Descartes' method, Newton spoke highly of Omerique's way of recovering the method of analysis from the ancient Greeks. In Newton's own words:

> I have looked into Omerique's *Analysis Geometrica* and I find it a judicious and valuable piece answering to its title, for it shows in the simplest way the method of restoring the Analysis of the ancients, which is more easily and readily for a geometer than the Algebra of the moderns. [...] In general he arrives to resolutions which are simpler and more elegant than those resulting from applying Algebra.[17]

[12] Ceva's result is found in *De lineis rectis se invicem secantibus; statica constructio* (Milan, 1678).

[13] In *Aperçu historique sur l'origine et le développement des méthodes en géométrie* (Bruxelles, 1837).

[14] J. Bernouilli has a demonstration of this result in *Opera Omnia* 1742, vol. 4, p. 33.

[15] In any quadrilater the sum of the squares of the sides equals the sum of the squares of the diagonals plus four times the square of the segment that joins the median points of these diagonals. *Variae Demonstrationes Geometriae* (1747).

[16] J. Zaragozà was born in 1627 in Alcalà de Xivert (Spain) and died in Madrid in 1679. He earned a Ph.D. in Philosophy from the University of Valencia and in 1651 joined the Jesuit order. He then went on to teach Rhetoric in Calatayud and Theology in Majorca, Barcelona and Valencia. Between 1660 and 1670 he lived in Valencia where he studied and taught Mathematics and Astronomy privately. Towards the end of 1670 he was offered the professorial Chair of Mathematics at the Imperial College in Madrid, where J. Charles de la Faille and C. Richard had taught earlier. He spent the remaining nine years of his life there teaching and writing. During this period he published *Geometria Magna in Minimis* (Toledo, 1674) and other works of a more didactical nature. Zaragozà also left a whole series of unpublished manuscripts on classical geometry. In Cotarelo (1935) there is a detailed biography of J. Zaragozà. Navarro Brotons (1985) mentions J. Zaragozà's contributions to astronomy. A. Dou (1990) places J. Zaragozà within the mathematical context of 17th c. Spain.

[17] Newton's letter to an unknown addressee that Pelseneer dates around 1699. See López Arnal (1992).

Thirdly, Jean Charles de La Faille published *Theorema de centro gravitatis partium circuli et ellipsis* in 1629, where for the first time he calculated the centre of gravity of a sector of a circle. De La Faille was disciple of the Belgium Jesuit geometer Saint-Vincent and he entered the Imperial College in 1629 to teach mathematics among other courses. In this work, La Faille established a formula to calculate the distance "d" between the centre of a circle and the centre of gravity of a circular sector. This distance can be given by the equation d = (2/3 R) (chord S) / (arc S), where R is the radium of the circle and S, the sector.

It is historically interesting to notice that La Faille believed that the calculus of centres of gravity was an important step towards the quadrature of the circle, which was the quest of so many at the time. Some years later, Zaragozà cited La Faille in his GMm when relating the calculus of the centrum minimum to the quadrature of the circle. La Faille's work was published before Guldin's one on centres of gravity and it was praised by Huygens in a letter to St. Vincent.

Finally, I shall refer to *Apollonii Pergaei Conicorum* by Claude Richard, who was the Spanish mathematician that most dealt with a topic very common at the time: the recovery and study of the eight Books of Apollonius' *Conics*. In 1655, taking as reference Commandino's Latin translation (1566) of the first four Books of Apollonius' *Conics*, Richard published a luxury edition in which he added notes including original lemmas and corollaries. In addition, Richard elaborated a conjectural restitution of the last four Books of Apollonius' *Conics*, which was never published but exists in manuscript version in the Academy of History of Madrid. Richard did such restitution when no version of the four last Books of Apollonius' *Conics* was available. The same topic was dealt with by the Italian geometer V. Viviani, who wrote a conjectural reconstruction of Book V in 1659. Three years later than Richard, the Italian geometer Borelli discovered in the Florentine Medicis' library a manuscript that contained an Arabian version of Books V, VI and VII of Apollonius' *Conics.*[18] As it is known, a complete version of the first seven Books was not published until 1710 by the English astronomer Halley.[19]

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[18] On the basis of this manuscript, Borelli as geometer and Abraham Ecchellensis as translator published an annotated version of these three Books in 1661.

[19] Halley based his work mainly on an Arabian version by Thabit-ibn-Qurra.

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