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## Leibniz in Paris

### (1) Introduction: The scope of readings

The extensive literature is dedicated to the Parisian period of Leibniz's life, the main attention being focused on his mathematical discoveries and the invention of the calculating machine. The other aspects of his creative activity are known sufficiently less. This paper presents an attempt to fill this gap to some extent; it is a by-product of the work on preparing Leibnizian scientific manuscripts written in 1672–1676 during his stay in Paris,<sup>1</sup> to publication. We will dwell mainly on manuscripts devoted to mechanics though it is worth mentioning that in this very time Leibniz has been interested in the more broad scope of physical problems, in particular in optics, astronomy, and in the vacuum technique. It should be added that in fact the variety of his interests was surprisingly great being far beyond the limits of scientific disciplines.

In this respect the manuscript LH035, 08,39, ff. 151r,v is very instructive. It contains the list of subjects that have been of interest for Leibniz. We find there not only mathematical and physical problems but also many exotic items which were not so easy to decipher.

For example, one of the items says: "Hookii tornus dioptricus. Scriptura coelestis, Gaffarelli et Bangi. Tachygraphia Anglicana". Here Leibniz refers first to the works of Robert Hook, on grinding optical lenses and then he mentions two different authors well known in the 17th century by their semi-mystical books, Jacques Gaffarel (see: Gaffarel, J. *Curiositez inouyes, sur la sculpture talismanique des Persians. Horoscope des Patriarches. Et lecture des Estoilles*. Rouen: 1634) and Thomas Bang (see his book *Caelum orientis et prisci mundi triade Exertationum Literarium Repraesentationem, curisque Thomae Bangi ... investigatum*. Hauntiae: 1657). The next item clearly means English stenography that was becoming popular at that time.

Another example: "Signatura rerum. Crollius". It is a reference to the book of German jatrochemist Oswald Crollius: *Oswaldi Crollii Basilica chimica. Pluribus selectis et secretissimis ... descriptionibus, et usu remediorum chym. selectissimorum aucta a Johanne Hartmanno*. Lipsae: 1634.

Leibniz further mentions other curious books coming back from time to time to his favourite subjects. Thus, in addition to Hooke's works on technical optics he says about "Wrenni hyperbola per tornum" without any doubt referring to a paper by Christopher Wren on hyperboloids and on methods of grinding glass hyperbolic surfaces published in 1669 (Wren, Ch. The generation of an Hyperbolic Cynindroid demonstrated, and application thereof to the grinding of Hyperbolic glasses. *Philosophical Transactions*, June 1669), and about "Huddenianis Lentibus, physico artificio tornatis", that is about lenses made by Dutch scientist Jan Hudde who has been known for his construction of a microscope with spherical lenses (1663) and for his collaboration with Spinoza for producing lenses for a telescope.

Among scientific books mentioning in the list we find the *Harmonices Mundi* by Johann Kepler, and it is worth to note that Leibniz is interested in the mathematical section of the book devoted to the so-called tessellations ("Kepleri pars harmonica de figuris"). This problem apparently is of great importance for him since he has placed it into a special item: "Elegantes formae, quas singulari quodam delectu vitrarii et pavimentarii sive tessellifices sola dispositione conciliant"; there is another Kepler's book in the list also purely mathematical, it is the *Stereometria doliorum*.

As far as the purely scientific problems are concerned, Leibniz is interested mainly in mechanics: the trajectory of a projectile and of a water jet, isochronism of a pendulum, the work of Archimedean

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<sup>1</sup> Niedersächsische Landesbibliothek, Hannover, Leibniz-Handschriften, v. 35, 37

cochlea, waves on the water surface spreading from the thrown stone; optics: the passage of light through the border of two adjacent media, the origin of rainbow in particular; cartography: methods of making maps and projections of the plane map onto a spherical surface, convex and concave; crystallography; magnetic properties of solids and more.

As to the subjects not belonging to science itself, the variety of his interests is really astonishing. Everything attracts his attention: lathes and grinding, looms and weaving manufacturing of silk stockings in particular, polishing of diamonds and other hard stones, problems of measuring and the measuring of Egyptian pyramids (here Leibniz refers to the book by English mathematician and egyptologist John Greaves: *Pyramidographia or a Discourse of the Pyramids in Egypt*. London: 1646), engraving, producing of typographic images, cards and chess, etc. As a separate item he also puts down the art of writing as well as the invention of the pen and methods of ciphering (“De complicatione literarum, deque modo ita complicandi, ut difficile sit aperire ignorantibus, sine ullo sigillo”).

In all this variety of the subjects mechanics had a special place. When Leibniz came to Paris he was a novice in mechanical problems as he confirmed it himself on the margin of his summary of Galilean *Discorsi*: “cum ista scriberem eram in his novus”. And with the passion of a newcomer he started to make up for the lost time. It was known that Huygens at the meeting in the autumn 1672 advised him to read mathematical works of Wallis and Saint Vincent, and probably the literature mentioned by Huygens contained the more extensive list of works, which would be necessary to read. Similar information Leibniz acquired from the constant personal contacts with his colleagues and from the vast correspondence. The great part in this matter played his visit to London and the acquaintance with Henry Oldenburg, Secretary of the Royal Society.

In the first years of his stay in Paris, Leibniz apparently was busy mainly with mathematical problems as well as with his invention of the calculating machine, and he really turned to the study of mechanics somewhere between the end of 1673 and the beginning of 1675. In any case the above-mentioned quotation belongs to the manuscript of the first half of 1675. In another manuscript dated by August 1673 and dedicated to the problem of the finding of the centre of gravity (also in connection with the *Discorsi*) he confirmed it once more: “Haec cum scriberem nondum intelligebam quid esset centrum gravitatis.” The study of manuscripts collected in the volume 35 and 37 of Leibniz’s manuscripts showed that he very attentively read three books: *Discorsi* by Galileo, *Mechanica* by Wallis, and *Traité de la percussion ou choc des corps* by Mariotte.

Among other books constantly mentioned in his manuscripts it is worth noting *Mechanica Graeca* by Aristotle, *Hydrostatical Paradoxes and New experiments Physico-Mechanical* by Boyle, *De resistentia solidorum* by Alessandro Marchetti, *La Statique* by Pardis, *Abrégé des dix livres d’architecture de Vitruve* by Perrault (here Leibniz was mainly interested in the section on simple machines), *Traité de mécanique* by Roberval, and *Traité de physique* by Rohault.

The acquaintance with this literature was necessary for analysis of three major subjects which transformed afterwards into his first scientific papers on mechanics. Thus, the study of Galilean *Discorsi* gave him the impetus for the later article “Demonstrationes novae de resistentia solidorum” published in *Acta eruditorum* in 1684. In the manuscripts of the Parisian period Leibniz already pointed to the mistakes of Galileo in the solution of the problem of the fracture strength of a beam. Later on he revised his earlier ideas more correctly. Galileo considered a beam as an absolute solid, which is broken in the place where the rupture stress exceeds the limit. In fact however no body is an absolute solid and we have to take into account its elastic and plastic properties. It was the reason why the calculations of Galileo did not comply with the later experiments made by Mariotte. And Leibniz in his article of 1684 introduced the elastic tension in order to meet the Mariotte’s data.

Another major problem was the motion in resisting media. Some manuscripts of the Parisian period show that the basis of his theory has been laid in that very time; twelve years later these ideas have been transformed into his paper “Schediasma de resistentia medii” (*Acta Eruditorum*, 1689).

The third problem concerned the possibility of perpetual motion and perpetuum mobile. In Paris Leibniz did not still come to a conclusion of impossibility to realize a perpetual motion and moreover, he was trying to work out a design which would be able to accomplish it.

## (2.1) Perpetuum mobile with magnets

Leibniz's interest to the problem was quite natural: the 17th century abounded in attempts to construct such a mechanism, Leibniz as it followed from his manuscripts did not cease to be interested in this subject (even in the 80's!) and while being in Paris he presented and discussed two variants of it what might be surprising to historians of science since afterwards, when Leibniz came to the more profound understanding of the conservation laws, he abruptly denied the possibility of existence of the perpetuum mobile.

The description of the first design we find in the manuscripts LH037,05,ff.59v, 58r,v. We cannot definitely say that this construction is his original invention for there were a great number of similar machines at that time, and probably he borrowed the idea from someone else. In any case the design (fig.1) is the following: there is a wheel  $ABCD$  which can rotate around its centre  $A$  in the vertical plane. On two perpendicular diameters  $BD$  and  $CE$  there are four round holes, and the fifth hole is in the centre, all of these holes being of the same size. Between the holes, along the corresponding diameters Leibniz places glass tubes  $FG$ ,  $HI$ ,  $KL$ , and  $MN$  are filled with a liquid and hermetically sealed; a steel ball can freely move inside each tube, practically without friction.

The initial position of these four balls (they are located in the points  $F$ ,  $I$ ,  $L$ ,  $M$  correspondingly) resulted into the clockwise rotation. After a quarter of revolution has been made it is necessary for continuous rotation that the disposition of the balls would be the same as at the very beginning. For this purpose Leibniz places a magnet suspended to the horizontal diameter of the corresponding hole providing that the diameter itself can freely slide within the hole.

Leibniz first supposes that all magnets have the same force, and when the tube  $IH$  is in the vertical position, the ball in it is attracted by the extreme magnet  $O$ , which should have removed it from the central magnet  $Q$  because the distance  $HQ$  is bigger than  $OH$ . And vice versa: the ball in the vertical tube  $MN$  is attracted by the central magnet  $Q$ , which should have removed it from the extreme magnet  $E$  because the distance  $EN$  is bigger than  $NQ$ .

Then Leibniz says that if the machine is in the position shown on the drawing, the balls located in  $I$  and  $M$  will acquire an impetus for the motion from  $C$  to  $D$ . Therefore, the part  $CDE$  will overweight the part  $EBC$  since the balls  $M$  and  $F$  are closer to the centre than the balls  $I$  and  $L$ , all other things being in equilibrium; then the ball  $I$  will move to  $N$ , and when the magnet passes from the position  $C$  to the position  $D$  (much more distant from it) it will not slide down back to  $H$  because of "a certain curvature of the tube and because it might happen that the distance  $LP$  is not larger than  $HO$ " (fig. 2). Further, the ball  $I$  moves to  $N$  and from there it will be attracted to  $M$ ; then it will pass to  $H$  and will be attracted to  $I$ , and so on. Thus the initial disposition of balls is being restored every time, and "due to its own motion the machine will come to a position that has been given to it by the initial impetus, and therefore the motion will continue".

"I confess, adds Leibniz here, that this invention is most ingenious and magnificent but nevertheless I have eventually found that something is lacking in its foundation." [It is the same phrase that makes us to suggest that Leibniz has not been author of this invention.] "I maintain that the middle magnet, which might seem to support the perpetuity of the motion, in fact destroys it by a malicious compensation."

Leibniz assumes (fig.3) that since the magnet  $Q$  does not move, the distance  $M'Q$  between the magnet and the ball will increase during the transit of the tube  $MN$  from the vertical to the horizontal position, and the ball in the tube will roll down back.

It should be noted here that this case is analogous to the case of rotation of the tube  $IH$  from  $C$  to  $D$  already discussed by Leibniz. Like in the first case we could say that the ball would not slide back due to "a certain curvature of the tube" as well as due to the fact that the distance  $FQ$  always could be made shorter than  $QN$ .

Leibniz however prefers to ignore such reason. Instead, he begins to analyze what will happen if we put magnet  $Q$  in the very centre of the hole, or if we make the length of the tube greater; another variant is to increase the size of extreme holes and, at last, he discusses what will happen if we make the central magnet weaker than extreme ones. Leibniz comes to a conclusion that all these changes do not work: the ball continues to roll down back and the rotation ceases.

It is interesting to note that in this discussion he once mentions friction as an obstacle to the motion: “the necessity of the magnets to change the place will considerably hinder the motion [for] in such a way they are continuously rubbed against the axes.”

In final part of the manuscript he deduced a formula by means of which we can find the proper diameter of the extreme hole providing that the force of magnet is inversely proportional to the distance:

$$h = \frac{b}{\frac{a}{c-d} - i} \text{ or } h = \frac{\left(\frac{a}{c-d} + f + g\right)(c-d-e)}{\frac{a}{c-d} - i}$$

where (see fig. 4):  $h$  — diameter of the hole,  $a$  — force of the central magnet,  $b$  — force of the extreme magnet,  $c$  — length of the tube,  $c-d$  — radius of the central hole,  $e$  — distance between the upper magnet and the ball located in the lowest position,  $g$  — an excess of the attractive force of the upper magnet in comparison with that of the central magnet (for the ball located in the lowest position),  $i$  — an excess of the attractive force of the lower magnet in comparison with that of the central magnet (for the ball located in the lowest position).

Having received this formula Leibniz just says that the greater  $h$  is the stronger will be the swinging of the magnets, that is the friction that will hamper the rotation. Therefore, he concludes, there is no doubt that everything at last will come to equilibrium. And this is not astonishing because two efforts of attraction are here, one to the centre of the earth, the other to the magnet, but both of them are continuously acting so that it is impossible to do nothing and to proceed not even in the least and nevertheless to act.

Leibniz was clearly dissatisfied with this machine with magnets, and in the next manuscript he transformed it to construct a purely mechanical device.

## (2.2) Perpetuum mobile with elastic elements

The manuscript LH037,05, ff.57r,v contains the description of the machine. It begins with the basic principle of constructing perpetuum mobile:

the whole artifice of perpetual motion consists in finding the way of restoring the restoring force without using the force, which has to be restored. For that reason two forces have to be connected to each other in such a way that the restoring force acts separately whereby everything is compensated without affecting the machine. But this can happen in an admirable way.

The design suggested by Leibniz is the following (fig. 5): there is a tube  $AB$  divided in the middle  $C$  in two parts by an impenetrable partition. Within each part of the tube  $AC$  and  $CB$  a heavy ball can freely move. At the initial moment, the tube is placed in the horizontal position, the left ball being located near the centre  $C$  and the right ball — near the end of the tube  $B$ . It is clear that in this case the tube will begin to rotate clockwise, and when it comes to the vertical position, an elastic mechanism hidden in the tube will raise the ball upwards. If the tube crosses the vertical line, the motion will continue.

Leibniz does not give the description of this elastic mechanism in detail but gives some hints how it can be constructed. In the middle of each half-tube he suspends a weight ( $H$  and  $G$  correspondingly) on a rigid pivot. While the tube is rotating from the horizontal to position to the vertical, every weight will also rotate but in respect to the tube and wind the spring connected with it. When the tube comes to the vertical position, the spring is released and drives a ball up. Thus both balls are simultaneously fixed in this new position.

Leibniz further says that the spring could be released a little bit earlier than the tube crosses the vertical line; then “everything will be as earlier but the initial disposition will be replaced by the opposite. And this reciprocation will be repeated without an end.”

And if is desirable to get a continuous rotation in one direction, Leibniz suggests to join the second analogous tube perpendicularly to the first one, or even better — to arrange several tubes at the same

axis. Thus, in spite of the fact that Leibniz did not give the details of the design, he apparently believed that the machine would really work.

### (2.3) Perpetual motion: The problem of storing energy

The third manuscript (LH037,05, ff. 92r,v; 93r) dedicated to the perpetual motion has an instructive title: A regular continuous motion due to irregular discontinuous agent, or the perpetual wind-clock (fig. 6). Here, Leibniz is trying to get a continuous motion by means of energy that is previously accumulated (for example, the potential energy of a lifted weight), providing that this energy is of natural origin (for example, the energy of wind). The idea of the machine is the following: Let us have a windmill that elevates (when the wind blows) a certain weight up to a certain height (for example it serves as a weight in a pendulum clock), and while it is sinking the windmill elevates another weight similar and equal to the first, to the same height. To realize this idea it is necessary to make the period of time during which the first weight is sinking from the upper point to the lower sufficiently great so that for this time the second weight can be elevated from the lower point to the upper, and it is necessary to invent a mechanism for the weight exchange.

As far as the time is concerned, the problem is easy to solve, a week clock movement is a common thing, and one can be sure that in a week the wind will be able to lift the second weight to the proper height. Also Leibniz gives us some hints how the exchange mechanism should be constructed.

The most interesting part in the whole design is its kinematical part, namely a device that transforms the rotation in every direction into the rotation in one direction. The point was that Leibniz rejected the commonly used design of a windmill with the oblique fixed wings in favour of strictly vertical wings. Let us have, — he writes, — the wheel  $abcd$  in the horizontal plane, which is half-opened and half-closed. The opened side is  $abc$  and the closed —  $adc$ . The wings attached to it along the circumference are fixed in the points  $a, e, b, f, c$ , etc. and rotated by the wind.

The next passage is especially curious:

One of the sides is closed for the wind cannot produce its own work blowing simultaneously into opposite wings. For example, if the northern wind (which is blowing from  $b$  to  $d$ ) simultaneously acts onto the wing  $a$  and onto the opposite wing  $c$ , so that the equilibrium there would emerge and there would be no reason for the rotation in any direction. In such a way a watermill is half-submerged only, otherwise it cannot be rotated by a current stream or it moves too slow. Thus I am wondering why we does not observe this at the work of common windmills.

Let this wind-wheel has the axis  $gh$  which rotates together with the wheel  $abcd$  and moves the cogged wheel  $ilmn$  providing that it joined the cylinder  $mn$  which has the cogged discs at both of its ends. However, if we construct such a transmission, it will not work because the cogged discs at the ends of the cylinder (firmly connected to it) must be rotate in opposite directions when the cogged wheel rotates. Therefore the transmission should be supplemented by a device that makes the rotation possible, and Leibniz introduces such a device analogous to the modern ratchet. The cogs of these discs, he says, should be made in such a way (fig. 7) that they can move only in one direction, and after they have been deflected and let the cogs of the wheel to pass, they become straight again. For example, the cog  $op$  is moving around the centre  $o$  only to  $q$ , not to  $r$  due to the obstacle  $r$ , and if it is moved to  $q$  by some force, when it ceases to act, the dent will fall in the previous perpendicular position, by natural gravity.

We can reconstruct the design of the transmission (fig. 8); here the cogged wheel  $W$  is joined to the cylinder  $C$  by means of cogged discs  $A$  and  $B$ , which are firmly fastened to the cylinder, and each disc is provided with above mentioned ratchet. It is easy to see that if the wheel  $W$  rotates clockwise, it will catch the left disc  $A$ , while it will pass freely through the ratchet of the right disc  $B$ , therefore both discs,  $A$  and  $B$  together with the cylinder  $C$  will rotate clockwise (if we look at them right to left). And vice versa: if the cogged wheel rotates counter-clockwise, the disc  $B$  will be the working one and the disc  $A$  the idle; as a result the cylinder will rotate clockwise (if we look at it from right to left) again.

Therefore, Leibniz concludes, the cylinder will always rotate in the same direction notwithstanding the rotation of the winged wheel.

In the remaining part of the manuscript he briefly describes the mechanism for the weight exchange: the principle part of it is presented by two chain transmissions, the first is a part of the clock, the second — of the windmill. There are two weights; when one is sinking, another is being elevated; the exchange takes place when the first weight reaches its lower point (the depository). At this very moment the second weight reaches its upper point and happens to be suspended on a special hook. As soon as the first weight touches the depository, an elastic element (or a spring) will release the second weight from the hook and it will pass to the first chain transmission. Simultaneously the second hook will catch the weight stored in the depository and start to lift it up. Leibniz does not give details concerning this “elastic mechanism”.

This manuscript is remarkable not only due to Leibniz’s persistency in constructing the perpetuum mobile or to the ingenious transmission construction, but also because he nearly first time in the history of science presented the idea of the storing of energy.

### (3) The motion in resisting media

The manuscripts LH035,IX,11, ff. 1r–13r; LH037,V, ff. 4r–12v written in 1675 are dedicated to the problem of motion in resisting media. It is well known that in 1689 Leibniz presented his results in the article entitled “Schediasma de resistentia medii” and published in the January issue of the *Acta eruditorum*. When Newton got acquainted with this article (as well as with other two appeared in the same journal), he decided Leibniz had used there his own results published in the *Principia*. Later he wrote about it:

In the *Commercium Epistolicum*, mention was made of three Tracts written by Mr. Leibniz, after a Copy of Mr. Newton’s *Principia Philosophiae* had been sent to Hannover for him; and after he had seen an *Account* of that book published in the *Acta Eruditorum* for January and February 1689. And in those Tracts the principal Propositions of that Book are composed in a new manner, and claimed by Mr. Leibniz as if he had found them himself before the publishing of the said Book. But Mr. Leibniz cannot be a Witness in his own Cause. It lies upon him either to prove that he found them before Mr. Newton, or to quit his claim.<sup>2</sup>

In 1688 Leibniz wrote about it in the letter to Otto Mencke:

Conclusions about the resistance of the medium which I have put on a special sheet, I had reached to a considerable extent twelve years ago in Paris.<sup>3</sup>

In fact, in October 1687 Leibniz left Hannover with an important diplomatic mission for Italy and lived there about two years; he received Newton’s book in the middle of April 1689, two months later after his paper had been published in the *Acta eruditorum*. Without any doubt the *Principia* happened to give an impetus for his own work, because when Leibniz read the review of this book in another issue of the *Acta*, he hurried up to complete the investigations which he had started many years ago, the study of motion in a resisting medium including.

Eric Aiton, in his paper<sup>4</sup> of 1972, analyzed the “Schediasma de resistentia medii” in detail and proved on the basis of the manuscript drafts of this article that the leading method in this investigation was essentially infinitesimal. As a result he rewrote the Leibniz’s propositions and their proofs using the mathematical symbolism and demonstrated the adequacy of mathematical (in the manuscript drafts) and verbal (in the published paper) statements. Nevertheless the question whether Leibniz got his results “twelve years before in Paris” still remains open.

It is possible to answer this question through a detailed analysis of some manuscripts of April, May and December 1675 dated by Leibniz himself. First of all, however, it is worth adding an

<sup>2</sup> “An account of the book entitled *Commercium epistolicum*...”, *Philosophical Transactions* (1714) 29, p. 208.

<sup>3</sup> *The Correspondence of Isaac Newton*. Cambridge, 1961 v. III, p. 5.

<sup>4</sup> E. Aiton. “Leibniz on motion in a resisting medium”, *Archive for History of Exact Sciences* (1972) 9, pp. 257–274.

important remark. Like in the case of the invention of calculus (as we remember the leading notion for Newton was velocity while for Leibniz it was a characteristic triangle) in the case of resistance the initial approaches to the problem were different though the final results are identical. Newton understood the resistance of a medium as a kind of pressure at motion, while for Leibniz the main attention was focused on the interaction between the body and every single particle of the medium. It was the resistance of this particle to the mobile that he called absolute resistance. In the introductory part of his manuscript “Du frottement”<sup>5</sup> Leibniz said that previous scholars tried to find the time-acceleration dependence while he believed that for studying the problem of friction it might be more useful to have the dependence between acceleration and the distance traversed.

On the one hand, Leibniz saw no difference between the resistance mechanisms for the motion of a solid along the surface of another solid and for the motion of a solid in a liquid medium. In both cases the physical cause of the resistance is the friction of particles. In Parisian manuscripts he proposes different models of such resistance:

When one body is moving along the other with a certain difficulty it is possible to imagine some points or obstacles on the surface of this second body which resist to the motion of the first; this obstacles can bend down and than draw themselves up, and one can represent this effect mechanically by means of a cog or a tooth on a hinge which can bend and restore its position due to some springs or levers.<sup>6</sup>

One type of resistance he calls absolute and claims that it is not dependent on the velocity of the moving body; on the other hand, he stressed the existence of another kind of resistance, the respective resistance which depends on the impact of fluid on the body the more greater the stronger this force of impact is; in this case he believes that the resistance is proportional to the velocity. Twelve years later Leibniz worked out the correct and clear explanation for both types of resistance in his “Schediasma de resistentia medii” but in 1675 he had already possessed these notions to a great extent.

Eric Aiton in the above-mentioned paper explained in detail their essence having concluded that

Leibniz’s theory of absolute resistance in fact corresponds to Newton’s theory of resistance proportional to velocity and his theory of respective resistance corresponds to Newton’s theory of resistance proportional to the square of velocity.

For the present discussion it is important to stress the point that the absolute resistance as Leibniz understood it is a characteristic of the individual particle; if a body moves on the surface of another body, the total action of all such particles, which the body encounters along its path, will be proportional to its velocity, that is, to the distance traversed. It is the reason why Leibniz is seeking the dependence between distance and velocity.

Let us turn now to Leibniz manuscript. He begins his analysis of absolute resistance with Theorem I: “A body, the motion of which is uniform is slowing down by every element of place that it passes, the remaining velocities being between themselves as the distances which are left to traverse.” Keeping in mind the above-mentioned remarks, it is possible to reformulate it: If the motion of the body is uniform and it is retarded by the resistance proportional to the velocity, velocities will be as distances which are left to traverse. The corresponding Theorem I of the Second Book of the *Principia* say: “If a body is resisted in the ratio of velocity, the motion lost by resistance is as the space gone over in its motion.”<sup>7</sup> Let us show that these two theorems are equivalent.

While proving this proposition Leibniz does not use a mathematical symbolism constructing his demonstration by the simple geometric reasoning: fig.9 gives the dependence of velocity  $v$  as a function of distance  $x$ . It is clear that triangle  $abc$  is similar to triangle  $v_0 O X$ , therefore:

$$-\frac{dv}{v_0} = \frac{dx}{X},$$

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<sup>5</sup> LH035,09,11, ff. 3r–4r

<sup>6</sup> LH035,09,11, f. 10v

<sup>7</sup> I. Newton. *Principia mathematica*, A.Mott, trans. London, 1934, p. 235

where  $X$  – the maximum distance traversed. Then, we will get by integration, for any couple of values  $(x_1, v_1)$  and  $(x_2, v_2)$ :

$$\frac{v_1 - v_2}{v_0} = \frac{x_2 - x_1}{X}.$$

Since initially  $v_1 = v_0$ ,  $x_1 = 0$ , what gives:

$$\frac{v_0 - v}{v_0} = \frac{x}{X}, \text{ or } v_0 - v = \frac{v_0}{X} x, \text{ or } v = \frac{v_0}{X} (X - x).$$

This last formula is the expression of Leibniz’s Theorem I.

The proof of Newton’s Theorem I will be the same. Supposing the mass is equal to unity, we get the principal equation in the following form:

$$\frac{dv}{dt} = -k \frac{dx}{dt},$$

where  $k$  is a coefficient of proportionality. By integrating this equation and using the initial conditions as in the first case, we get

$$v_0 - v = kx$$

In order to determine the coefficient of proportionality  $k$  remember that in the end of a distance  $x = X$  and  $v = 0$ , therefore

$$v_0 = kX.$$

If we substitute this value in the previous equation, we will have:

$$k = \frac{v_0}{X} \text{ and } v_0 - v = \frac{v_0}{X} x.$$

Thus, Newton’s Theorem I is proved, and simultaneously its equivalency to Leibniz’s Theorem I.

Now let us turn to Theorem II of the Second Book of the *Principia*:

If a body is resisted in the ratio of its velocity, and moves, by its inertia only, through a homogeneous medium, and the times be equal, the velocities in the beginning of each of the times are in a geometrical proportion, and the spaces described in each of the times are as the velocities<sup>8</sup>.

In Leibniz’s manuscript “Du frottement”, a lemma and four theorems correspond to this Newton’s proposition.

Lemma states that increments of time for every element of space are inversely proportional to the velocities that the moving body has. Leibniz believes that since for sufficiently small intervals of time and for equal intervals of space we can assume that, the inverse proportionality between increments of time and velocities follows immediately.

Then Leibniz proves his Theorem II: the increments of time elapsed is inversely proportional to the spaces still to be traversed. Indeed,

$$v = \frac{v_0}{X} (X - x)$$

according to Theorem I and

$$dt = \frac{dx}{v}$$

according to the previous Lemma, hence

$$dt = \frac{X}{v_0} \cdot \frac{dx}{X - x},$$

i.e. the increments of time are inversely proportional to the distances still to be traversed.

<sup>8</sup> *Op. cit.*, p. 236.

The next Theorem III (fig. 10) says that the increments of time are the ordinates of a hyperbola with the asymptote EA and the centre in A, the abscises of which are the spaces still to be traversed.

This statement is easy to demonstrate: in fig.10 the curve GD(D)Q is given; if the ordinates of this curve, (B)(D), BD are the increments of time, they are (according to Theorem II) inversely proportional to the distances A(B) and AB; in its turn that means that the areas of rectangles A(B)(D) and ABD are equal. The equality of the said areas is the characteristic of a hyperbola, and since this construction is valid for the arbitrary chosen points (B) and B, the curve GD(D)Q is a hyperbola.

Theorem IV clearly follows the Theorem III: since the increments of time are the ordinates of a hyperbola, the times elapsed will be presented by the sums of the said ordinates, that is, by the areas of this hyperbola.

Theorem V states at last that if the distances still to be traversed by the moving body are as numbers, the corresponding times elapsed will be as logarithms. If:

$$dt = \frac{X}{v_0} \cdot \frac{dx}{X-x} \quad (\text{according to Theorem II}),$$

then we will get by integration:

$$\frac{v_0}{X} t = \ln \frac{X}{X-x} \quad \text{and} \quad x = X(1 - e^{-\frac{v_0 t}{X}}).$$

It is obvious that  $X-x$  are the distances still to be traversed, and therefore they will be as logarithms. It can be also seen that the distances represent a geometric progression because Leibniz (see above) chooses the exponential function as the abscissa for his hyperbola; in this case the “hyperbolic parts” (as Leibniz calls them), that is, the times will be equal. Therefore the theorem is proved.

Leibnizian Theorems II–V are equivalent to Theorem II of Newton: Indeed, according to Leibniz (Theorem V):

$$x = X(1 - e^{-\frac{v_0 t}{X}})$$

and the differentiation immediately gives:

$$v = \frac{dx}{dt} = v_0 e^{-\frac{v_0 t}{X}};$$

this expression is non other than the statement of Newtonian Theorem II: if... “the times be equal, the velocities in the beginning of each of the times are in a geometrical proportion”

It is necessary to explain here that Newton (like Leibniz) considers the exponential function as the limiting case of the geometrical progression and presents the exponential scale along the abscissa axis.

The similarity of their approaches is clearly seen from the corollary to the Newtonian Theorem II. Like Leibniz Newton examines here (fig.11) the hyperbola  $BG$  with asymptotes  $AC$  and  $CH$  and claims that the time of motion will be represented by the area  $ABGD$ . Besides, the input equation of Theorem II can be written as:

$$\frac{dv}{dt} = -kv,$$

$$\text{then } v = v_0 e^{-kt} \quad \text{and} \quad x = v_0(1 - e^{-kt}).$$

Thus we get the similar expressions for the distance traversed, both of Newton and Leibniz, they differ only by coefficients because Newton measures the time and the distance along the same interval AC.

Therefore the results obtained by Leibniz and Newton seems to be identical, and it proves the fact that Leibniz has got his theorems during his stay in Paris quite independently.

Being based on these materials it can express some views concerning the problem why Leibniz afterwards denied the idea of perpetuum mobile as well as the possibility of its realization in principle. We have seen that friction according to Leibniz is a result of numerous collisions of the body with individual particles of the medium. He seems to have also understood that in this process the energy is irreversibly lost. When Leibniz came to the proper understanding of the essence of conservation laws he simultaneously understood that the perpetuum mobile is impossible to construct because it is impossible to get free of friction.

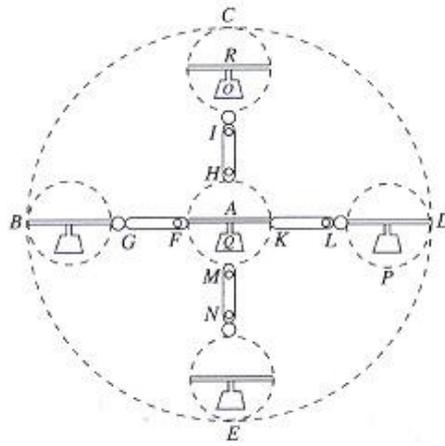


Fig. 1

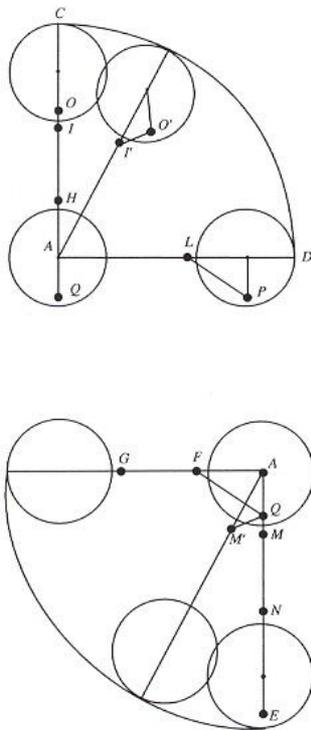


Fig. 2, 3

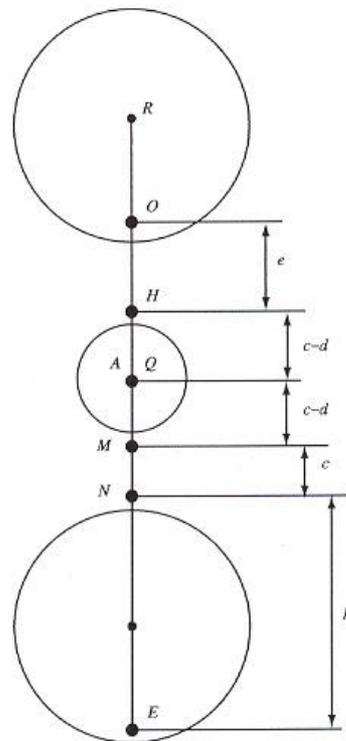


Fig. 4

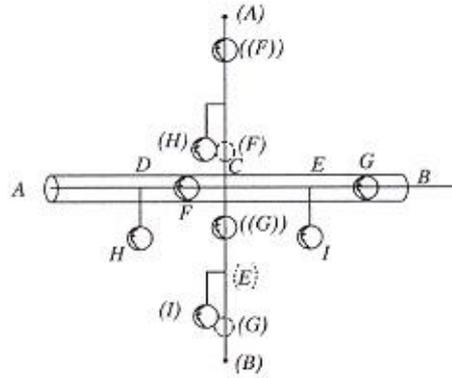


Fig. 5

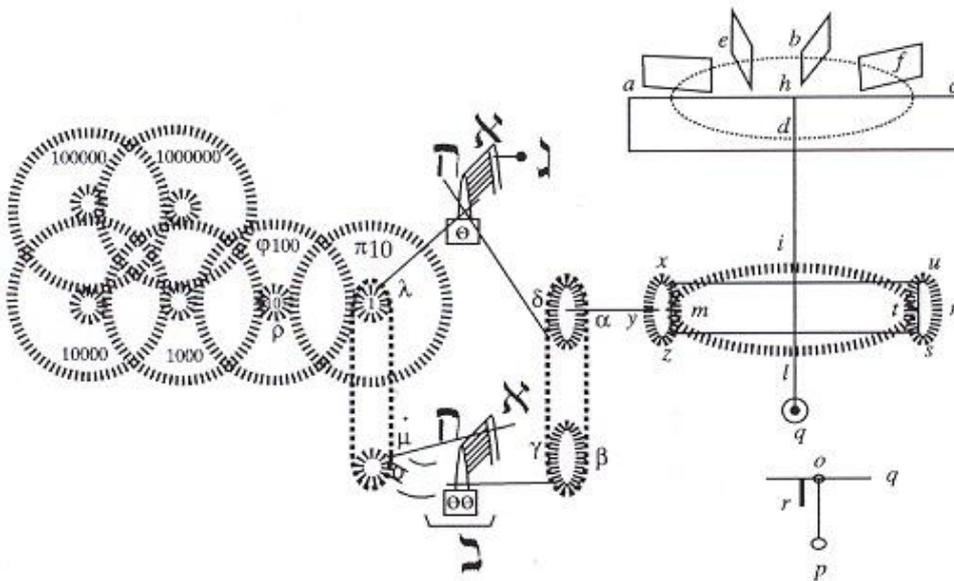


Fig. 6

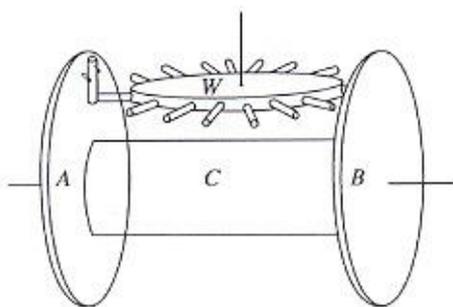
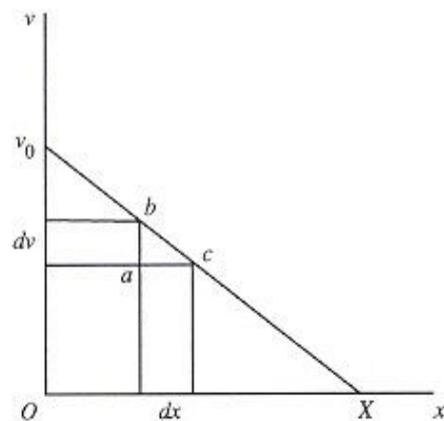
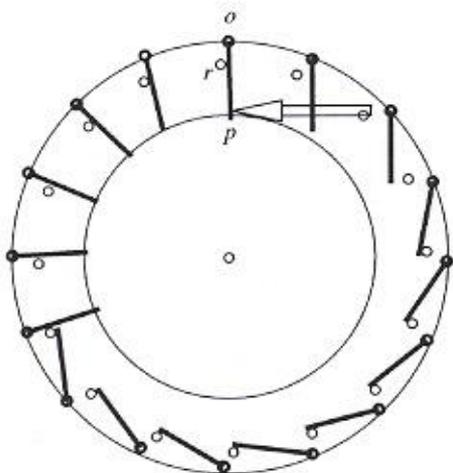


Fig. 7, 8

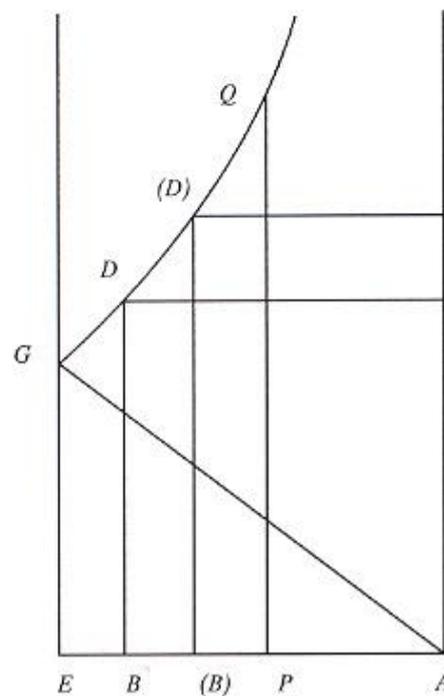


Fig. 9, 10

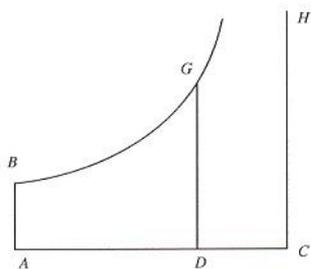


Fig. 11