

**Peter Barker \***

## **Towards a cognitive history of the Copernican revolution \*\***

### **(1) Introduction**

For several years I have collaborated with Hanne Andersen of the University of Aarhus in Denmark, and Xiang Chen of California Lutheran University, on studies of conceptual change in the history of science drawing on ideas from cognitive science, and especially drawing on theories of concepts developed recently in cognitive psychology. Much previous work in the history and philosophy of science talks about conceptual structures. The great advantage of these ideas from cognitive psychology is that they allow us, perhaps for the first time, to say in an empirically defensible way exactly what a conceptual structure looks like, and to make possible detailed comparisons of conceptual structures. We have presented our ideas most fully in *The Cognitive Structure of Scientific Revolutions* (Cambridge University Press, 2006).

In this brief paper I will sketch a cognitive account of the Copernican revolution, a subject treated at length in our book.<sup>1</sup> I will focus on one aspect of the revolution: the nature of the equant problem in Ptolemaic astronomy and Copernicus's response to it. I will extend our presentation slightly by giving historically defensible numerical values for some key parameters. I will begin by giving a cognitive summary of the revolution. This way of presenting familiar historical material may seem rather strange at first, but perhaps I can explain enough to make at least some of it intelligible, and to connect it to several key historical questions.

Ptolemaic astronomy employed a conceptual structure that relied entirely on object concepts. Although the key concept of uniform circular motion embodied a number of variable parameters, the values of all these parameters were fixed and unvarying in each planetary model. A difficulty appeared, however, in the form of the equant, an auxiliary concept that divided the class of circular motions into two. The new class of motions rotated non-uniformly when viewed from their geometrical centers, and required a conceptual structure that was incommensurable with the structure for the original concept of circular motion.

Copernicus reformed astronomy by eliminating the equant, and the awkward conceptual structure that it required. He was able to conduct planetary astronomy using only uniform circular motions in the original sense. Like Ptolemy, he used only object concepts with fixed values for parameters. Although his parameters differed from Ptolemy's, they were consistent with the ranges of earlier values. No new kinds of values were required for Copernican astronomy. Copernican astronomy is therefore commensurable with a version of Ptolemaic astronomy that avoids the equant, but is superior to it. Ptolemaic astronomy without the equant is inaccurate; Copernicus achieves the same accuracy as Ptolemaic astronomy with an equant, but without introducing motions that are not uniform about their geometrical centers.

This account illuminates several historical questions. Why, for the first fifty years after the publication of *De revolutionibus*, was Copernicus generally regarded as a reformer of astronomy not a revolutionary? The continuity of his conceptual structure with Ptolemy's explains this response on the part of sixteenth century readers. So, was there a Copernican revolution and when did it happen? Cognitive arguments locate the beginning of the revolution at the moment when Kepler developed a new version of Copernican astronomy using event concepts like 'orbit' in place of object concepts like 'uniform circular motion'. This structure is incommensurable with both Ptolemaic astronomy and Copernican

---

\* Department of the History of Science, University of Oklahoma, Norman, OK USA; email: [barkerp@ou.edu](mailto:barkerp@ou.edu).

\*\* This paper is also published in *Organon*, vol. 35, 2006, p. 61–72.

<sup>1</sup> Hanne Andersen, Peter Barker and Xiang Chen, *The Cognitive Structure of Scientific Revolutions* (Cambridge: Cambridge University Press, 2006). See especially chapters 5 and 6. Subsequently referred to as *CSSR*.

astronomy as formulated by Copernicus. Hence, the general adoption of the structure introduced by Kepler (arguably at the time of Newton or later) marks the revolutionary break that separates ancient astronomy from modern theories.<sup>2</sup>

## (2) From equants to incommensurability

Most people are familiar with the simple version of Ptolemy's planetary models.

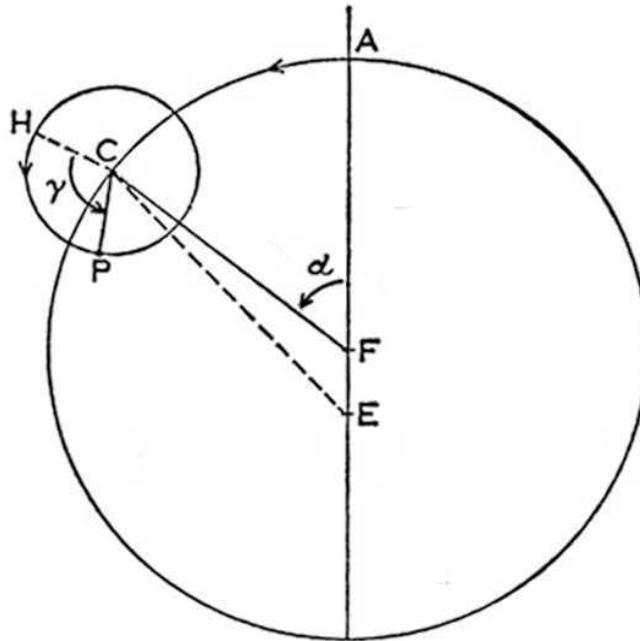


Figure 1: Ptolemaic model for the outer planets  
(simplified version)

KEY:

C = Geometrical center of epicycle (carried on deferent)

F = Geometrical center of deferent

E = Earth (position of observer)

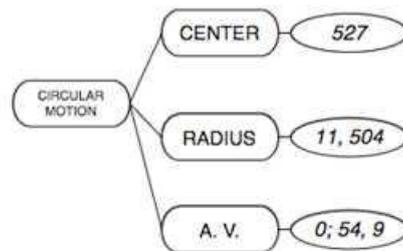
P = Planet (carried on epicycle)

alpha, gamma = Uniformly increasing angles

<sup>2</sup> I should emphasize here that this summary represents my best present understanding of the nature of the revolution from the viewpoint of cognitive studies of conceptual change, and that other historians might place a different emphasis on the same material. In a famous paper Otto Neugebauer questioned whether Copernicus had eliminated the equant at all (Otto Neugebauer, "On the Planetary Theory of Copernicus", in Arthur Beer (ed.), *Vistas in Astronomy*, 10 (1968) p. 89–103). Historical studies have also located the break between ancient and modern astronomy with Kepler (e.g. N. R. Hanson, "The Copernican Disturbance and the Keplerian Revolution", *Journal of the History of Ideas*, 22 (1961) p. 169–84) and highlighted the importance of the concept of an orbit (see below, note 9). Kepler introduced many innovations, not least his appeal to causes in astronomy proper. For a detailed historical consideration of the differences between Copernicus's position and Kepler's see Peter Barker, "Constructing Copernicus", *Perspectives on Science*, 10 (2002) p. 208–27. One benefit of the sort of cognitive analysis suggested in *CSSR* is that it provides clear criteria for evaluating conceptual shifts or innovations as revolutionary. From this viewpoint, Kepler's astronomy as whole represents a revolutionary break with the astronomical tradition, however his introduction of the event concept 'orbit' marks a second and even more substantive level of discontinuity with previous astronomical theories (*CSSR*, p. 151–61).

For example, in the case of the outer planets, a large circle or *deferent* carries a small circle or *epicycle*. Both circles rotate at constant speed, and the small circle carries the planet. One minor complication is the location of the observer, or the earth. This is not the geometrical center of the large circle, but a point well along one of the diameters, making the observer and the circle *eccentric*. Roughly speaking the motion of the large circle corresponds to the proper motion of a planet (that is its motion in longitude), and the motion of the epicycle generates the occasional stationary points and retrogressions, during which the planet briefly moves in the opposite direction to its usual motion.<sup>3</sup>

From the viewpoint of recent theories of concepts, both the proper motion and the retrograde motion in Ptolemy's models are explained using a single straightforward conceptual structure for uniform circular motion.



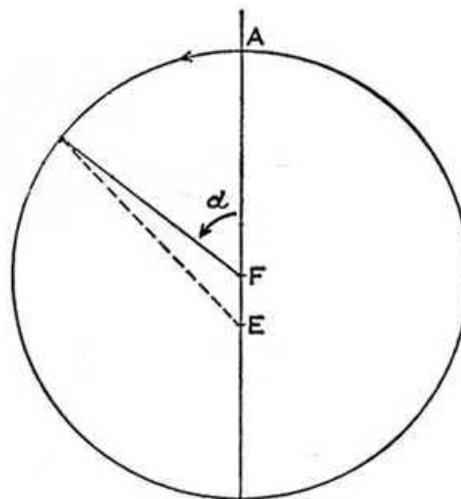
**Figure 2: frame representation for deferent motion of Jupiter (simplified version of Ptolemy)**

KEY:

CENTER = GEOMETRICAL CENTER earth radii

RADIUS = RADIUS OF ROTATION earth radii

A. V. = ANGULAR VELOCITY degrees per mean solar day



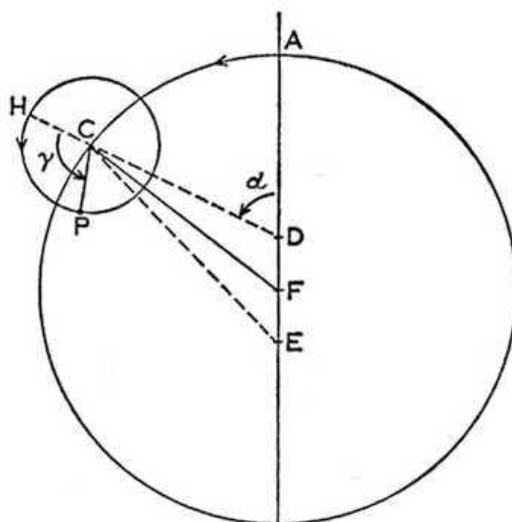
The concept of circular motion may be displayed as a frame in which three attributes stand out. The first attribute defines the position of the motion's center. A second attribute defines its radius. The third attribute defines the rate of rotation. There may be other attributes, as we will see below. Each attribute can take a range of values, shown as a set of nodes on the extreme right of the frame. Each circular motion in each model can be represented by a specific frame with the same attributes but different values. Thus we will need one frame each for the deferent and for the epicycle of each planet in the simplified version of Ptolemy we are now considering. In the frames for any given planet each attribute takes a fixed value, with different values giving models for different planets. To take a

<sup>3</sup> Ptolemy, *Almagest* IX, 3; Olaf Pedersen, *A Survey of the Almagest* (Odense: Odense University Press, 1974), p. 269.

concrete example consider the frame for the deferent motion of the planet Jupiter. The radius of its deferent is 11, 504 earth radii. The radius of its epicycle is 2, 205 in the same measure.<sup>4</sup>

In Ptolemy's planetary models the slowest rotation is the epicycle of Saturn, the fastest the deferent of the Moon. Although I have given these numbers in absolute measure, this is a slight distortion of history. The original parameters were specified as ratios with a deferent arbitrarily assigned the radius 60 for ease of calculation in sexagesimal arithmetic. The absolute distances could be calculated, using the same assumptions I have used, but the model is only used to calculate the angular position of a planet from some fixed direction. To make that calculation, only rates of rotation and relative sizes of circles are important.<sup>5</sup>

However, the simple deferent-plus-epicycle model is inadequate. We do not know precisely how or why Ptolemy introduced the equant but the best explanation seems to me to be the one offered by James Evans (1984).



**Figure 3: Ptolemaic model for the outer planets (including equant)**

KEY:

- D = Equant (center of rotation for center of epicycle)
- C = Geometrical center of epicycle (carried on deferent)
- F = Geometrical center of deferent
- E = Earth (position of observer)
- P = Planet (carried on epicycle)
- alpha, gamma = Uniformly increasing angles

Viewing the arcs in the sky that correspond to retrogressions predicted by the model, we are interested in correctly predicting two things: the direction in which a retrogression occurs, and its duration. Ptolemy seems to have considered two possible vantage points for viewing retrogressions: the eccentric point and a symmetrical point, equidistant from the center of the deferent but on the opposite side to the eccentric. He called this point the *equant*. Viewing retrogressions from one of these points gets durations right but directions wrong. Viewing the same arcs from the other point gets directions

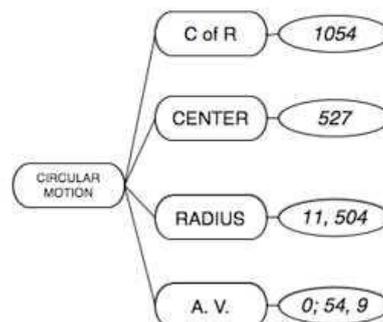
<sup>4</sup> Bernard R. Goldstein, "The Arabic Version of Ptolemy's Planetary Hypotheses". *Transactions of the American Philosophical Society* 57, 4/1967 (American Philosophical Society: Philadelphia, 1967). James Evans, *History and Practice of Ancient Astronomy* (New York: Oxford, 1998).

<sup>5</sup> J. Evans, *History and Practice of Ancient Astronomy*.

right but durations wrong. Ptolemy apparently realized that he could achieve accurate predictions by combining the two approaches. He places the observer at the eccentric point, and the second symmetrical point, the equant, is made the center of rotation for the epicycle; specifically the center of the epicycle, which is carried by the deferent, rotates uniformly about the equant, not the center of the deferent. The equant device thus has the curious effect of separating the center of rotation of the deferent circle from its geometrical center. A circular motion like the one now attributed to the deferent can also be represented as a frame, but it will evidently need an additional attribute.

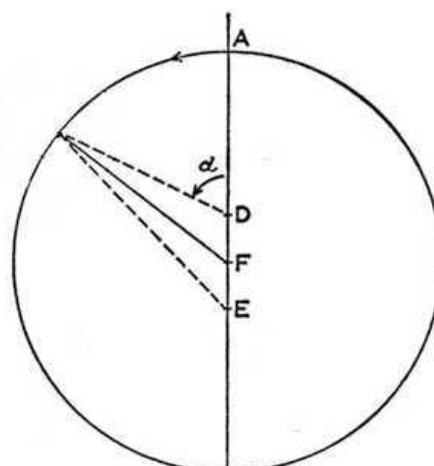
In the previous frame for circular motion, the same point functioned as the geometrical center and the center of rotational motion. As soon as the equant is introduced these two functions need to be identified as separate attributes. It now becomes an open question for any given circular motion whether these two points are distinct or coincident. The new frame we have drawn is clearly required to accommodate the full complexity of Ptolemy's planetary model, but these complexities make it incommensurable with the simpler frame considered earlier.

Incommensurability is a problem usually introduced by radical conceptual change, and marked by the appearance of new entities forbidden by the old conceptual structure. The best definition of incommensurability as it applies here is a restructuring of attributes and values in such a way that the new concept permits the existence of entities excluded by the old concept. This restructuring may involve addition or deletion of attributes, so one indication of incommensurability is that the frame for the new conceptual structure cannot be superimposed on the old one. However, in general, simple additions to existing conceptual structures will not generate incommensurability. Suppose, for example, that we add an attribute to represent the spatial orientation of the circular motion (defined as the angle between an axis perpendicular to the motion's geometrical center and some fixed line, for example the main North-South axis around which the heavens rotate).



**Figure 4: Frame representation for deferent motion of Jupiter (including equant)**

KEY:  
 C of R = CENTER OF ROTATION earth radii  
 CENTER = GEOMETRICAL CENTER earth radii  
 RADIUS = RADIUS OF ROTATION earth radii  
 A. V. = ANGULAR VELOCITY degrees per mean solar day



This does not change the ontology of circular motions, it merely tells us more about the circular motions we already have. However, the changes in the conceptual structure of the concept “circular motion” needed to accommodate the equant are not merely additive; they introduce a new entity that, in effect, is incompatible with and replaces the entities postulated in the older structure.

Now a modern mathematician might not regard the frame in Figure 2 as forbidding the existence of circular motions with different geometrical centers and centers of rotation, but at the very least we can see that there is no way of expressing such a configuration using that frame; we need the additional resources of the frame in Figure 4. Of course the type of circular motion allowed in the frame shown in Figure 2 can still occur if we use the conceptual structure presented in the frame from Figure 4, that is, the center of rotation and the geometrical center of the motion may coincide. But, it might be argued, these points remain distinct even when they coincide. So, in effect, the circular motions represented by the frame of Figure 2 are all replaced, after we adopt the frame of Figure 4, by motions with two centers that need to be specified. And we might add, this was actually true even before we recognized (or Ptolemy recognized) that it was possible to separate geometrical center and center of rotation in this way. So the new frame introduces a new ontology of circular motions that is incompatible with and replaces the old ontology.

Kuhn identified incommensurability between the conceptual structures of successive paradigms. In one of his best developed examples, Lavoisier’s chemistry is incompatible with pneumatic chemistry because it eliminates phlogiston, introduces oxygen, and more fundamentally redefines what counts as an element and what counts as a compound. But the two frames we have just considered lead to a curious outcome. Although not as dramatic as the Lavoisier case, there seems to be a difficulty of the same sort in Ptolemaic astronomy when we consider the introduction of the equant. Models that lack the equant embody conceptual structures that are incommensurable with the structures for models that employ the equant. Whether or not a modern mathematician would allow the former structures as special cases of the latter ones, the historical actors voiced clear objections to the equant’s separation of center of rotation and geometrical center. Copernicus himself says, in the first few paragraphs of the *Commentariolus*:

It seemed absurd [to our predecessors] that a heavenly body should not always move uniformly in a perfect circle. ... Yet [Ptolemy’s planetary theories] were not adequate unless certain equants were also conceived; from which it appeared that a planet never moves with uniform velocity either in its deferent or with respect to its proper center.<sup>6</sup>

And in the letter to the Pope which precedes the text of *De revolutionibus* he says:

... [Ptolemaic astronomers] admit many things that are seen to contradict the first principles of uniform motion.<sup>7</sup>

His chief disciple at the University of Wittenberg, Erasmus Reinhold, found this point so important that he inscribed it on the title page of his copy of *De revolutionibus*:

The Axiom of Astronomy: The motion of the heavens is uniform and circular, or composed of uniform and circular motions.<sup>8</sup>

---

<sup>6</sup> Leopold F. Prowe, *Nicolaus Copernicus* (Berlin: Weidmann, 1883–84). Vol. 2, p. 184–5; cf. Noel M. Swerdlow, “The Derivation and First Draft of Copernicus’s Planetary Theory: A Translation of the *Commentariolus* with Commentary,” *Proceedings of the American Philosophical Society*, 117 (1973) p. 423–512, here p. 433–4; E. Rosen, *Three Copernican Treatises: the Commentariolus of Copernicus, the Letter against Werner, the Narratio prima of Rheticus* (New York: Octagon Books, 1971) p. 57. A more accurate translation might be: “It seemed absurd [to our predecessors] that a heavenly body should not always move uniformly in a perfect circle. ... [Yet the planetary theories of Ptolemy] were not adequate unless certain equant circles were also conceived; from which it appeared that a planet never moves with uniform velocity either in its deferent orb or with respect to its proper center.” On the technical vocabulary highlighted here, see P. Barker and K.A. Tredwell, “The structure of the theorica orbs” (in progress).

<sup>7</sup> Nicholas Copernicus, *De revolutionibus orbium coelestium* (Nuremberg: Petreius, 1543) fol. III V, ll.12–13.

<sup>8</sup> Owen Gingerich, *An Annotated Census of Copernicus’ De Revolutionibus* (Leiden: Brill, 2002) entry I.217 (Edinburgh 1), p. 269.

The conceptual structures we have examined give us a simple way of representing these concerns; Copernicus, Rheticus and Reinhold wanted to use the simpler frame from Figure 2 and not the more complex frame from Figure 4 to represent circular motions.

### (3) Circular motion in Copernican astronomy

Let us next examine what Copernicus did with circular motions in his own models. To make my general point I will consider only Copernicus's model for an outer planet like Jupiter; the models for Venus and Mercury are progressively more complex, although the same general considerations apply.

To calculate the position of a planet from the viewpoint of an observer on the earth Copernicus employs three circles. The proper motion is now largely referred to an eccentric deferent, although this circle is eccentric to the mean sun rather than to the earth. Riding on this deferent is a small epicycle. This serves a completely different function in Copernicus's models. It does not generate retrogressions. Instead, when combined with a change in the eccentricity of the deferent, the small epicycle does the work done in Ptolemy's model by the equant. Finally, stationary points and retrogressions are handled by considering, in each model, a separate circle centered on the mean sun. We would call this circle the orbit of the earth — but of course the concept of an orbit was not introduced by Kepler until sixty-six years after the appearance of *De revolutionibus*.<sup>9</sup> In Copernicus's models, stations and retrogressions become line-of-sight phenomena caused by the earth overtaking an outer planet. Copernicus is also able to explain why retrogressions always occur when the sun, the earth and a planet form a straight line, and the difference in the sizes of the arcs of retrogression performed by different planets. Comparing Ptolemy with Copernicus, we could say that to explain retrogression Ptolemy introduces a circle carried by the eccentric deferent of each planet; Copernicus recognizes that this circle is actually the path of the earth viewed from the distance of the eccentric deferent.

From the viewpoint of conceptual structure, these changes make surprisingly little difference. The first and most important point is that Copernicus uses the frame from Figure 2 and not the frame from Figure 4 as the conceptual structure of circular motion throughout his models, that is, he abolishes the equant. He succeeds in performing all the tasks required by Ptolemy using only circles that rotate at constant speed about their geometrical centers. This, as we have already indicated, is Copernicus's chief announced motivation for his new system, and its main attraction to sixteenth century readers. Whatever unease there is about the introduction of the equant, or the tension between the conceptual structures of the frames in Figures 2 and 4, Copernicus removes it.

The second point is that Copernicus achieves all this without introducing any new combinations of attributes and values, or any new *kinds* of value, in the frames for circular motions that correspond to individual planets. According to Kuhn, variations in conceptual frameworks begin with changes in values; changes in combinations of values assigned to objects lead to the redefinition of attributes, or to the addition and deletion of attributes, which in turn cause conceptual change. It is important to see that none of this is going on in Copernican planetary astronomy.

There are three key attributes in the frame in Figure 2: center, radius and angular velocity. The angular velocities in Copernicus's models can be accommodated by the same ranges of values that were used by Ptolemy. The radii are different — the deferents become larger and the distance to the sphere of fixed stars becomes much larger. This may be grounds for complaint (Tycho Brahe famously objected that the Copernican cosmos contained "too much wasted space"), but increasing a distance is not a *conceptual* change. The most surprising thing is that the specification of center is also within the scope already allowed in Ptolemaic astronomy.

In Ptolemy's models (whether we use the frame from Figure 2 or Figure 4) the center of the deferent is *not* the center of the earth. The actual positions of the centers vary widely. To return to our example of Jupiter, the center of its deferent is located 527 earth radii from the center of Ptolemy's

---

<sup>9</sup> For the historical context of Kepler's introduction of the concept of an orbit, see especially, Peter Barker and Bernard R. Goldstein, "Distance and velocity in Kepler's astronomy", *Annals of Science*, 51 (1994) p. 59–73. and now Bernard R. Goldstein and Giora Hon, "Kepler's Move from Orbs to Orbits: Documenting a Revolutionary Scientific Concept", *Perspectives on Science*, 13 (2005) p. 74–111. For a cognitive appraisal of this episode see *CSSR*, p. 151–61.

cosmos, which places it inside the zone occupied by the planet Venus.<sup>10</sup> Note that in Ptolemy's system the Sun ranges between 1160 and 1260 earth radii, or just over twice the distance of Jupiter's deferent center. Copernicus ties his eccentrics not to a central earth, but to the mean sun. However he uses the same figure as Ptolemy for the distance between the earth and the sun. Hence, from the viewpoint of a sixteenth century reader, this is just the specification of another deferent center in a position not very different from those considered by Ptolemy. Notice also that all this can be considered without deciding whether the earth goes around the sun or the sun goes around the earth.

From the viewpoint of cognitive reconstruction, the most important issue here is whether Copernicus's planetary models introduce any new combinations of values, or edit the attributes, used in the frame for uniform circular motion. The evidence we have just considered shows that they do not. Copernicus may be seen as specifically excluding the additional attribute need to accommodate the equant in Ptolemy's account, and the values he specifies for positions of centers, radii and angular velocities all fall into ranges already established by Ptolemy. Admittedly Copernicus extends these ranges — as already mentioned his cosmos is larger than Ptolemy's. But the important issue here from the viewpoint of reconstructing cognitive structure is whether he needs to introduce any values in ranges of a completely new kind, or rearrange combinations of values in ways prohibited by the conceptual structure used by Ptolemy. Copernicus does neither of these things and therefore his conceptual structure might be adopted by a sixteenth century reader as a simple variation of a structure familiar from Ptolemy.

We can therefore imagine a sixteenth century reader of Copernicus entertaining the planetary models without feeling the need to adopt a moving earth or a stationary sun. And this is exactly what happened at Wittenberg, where Erasmus Reinhold led the way in adopting Copernicus's mathematical models, and thus avoiding the use of equants, while rejecting Copernicus's cosmology. At the same time that they were hailing Copernicus as the great restorer of astronomy, using his mathematical models and spreading his fame all over Northern Europe, people like Reinhold, his teacher Melanchthon, and his successor Peucer developed a canonical set of objections to Copernican cosmology, based on arguments from physics and separately on arguments from scripture. People who adopted Copernican cosmology remained extremely few in the period before Kepler introduced his alternative version of Copernicanism, including the concept of an orbit, in 1609 and Galileo offered his telescopic discoveries as a new support for Copernican cosmology in 1610.<sup>11</sup>

Let us consider, finally, the question of whether Copernican planetary astronomy is incommensurable with Ptolemaic astronomy. Notice that this is not a question about Copernican cosmology. You may adopt the planetary models in either a heliocentric or a geocentric version; the use of the mean Sun as the basis for the deferents of the planets does not commit you on the question of whether the Sun or the earth moves. The determination of incommensurability is made by examining the combinations of values and attributes used in the uniform circular motions of the different versions of planetary astronomy. And here we have suggested, first, that Ptolemaic astronomy including the equant is, in effect, incommensurable with a simpler version of the same astronomical models, which use only circular motions uniform about their geometrical centers. Of the two versions of Ptolemaic astronomy, with equants and without, Copernicus is incommensurable with the equant based version, and for exactly the same reasons that the simpler version of Ptolemaic astronomy is incommensurable with the equant based version. But Copernican planetary theory is *commensurable* with the simplified version of Ptolemaic astronomy that uses only the frame from Figure 2 as its concept of uniform circular motion. Copernicus was accepted as a reformer of astronomy not a revolutionary, because he gave sixteenth century astronomers what they had always wanted, a version of Ptolemaic astronomy that used the frame from Figure 2 for its concept of uniform circular motion, and not, as Ptolemy had done, the frame from Figure 4. Hence the revolutionary aspect of the Copernican revolution cannot be located in Copernican planetary astronomy.

---

<sup>10</sup> Ptolemy's Planetary Hypotheses gives Jupiter's mean distance as 11504 e.r. Taking the radius of the deferent as 60, Ptolemy expresses the eccentricity as  $2;45$  or  $2.75$ . Thus  $(11504/60) \cdot 2.75 = 527$  e.r. (correcting to the nearest whole number throughout).

<sup>11</sup> Katherine A. Tredwell, and Peter Barker, "Copernicus' First Friends: Physical Copernicanism from 1543 to 1610", *Acta Philosophica/Filozofski vestnik*, 25 (2004) p. 143–166.

In this paper I have given a short account of a story that is presented much more fully in a recent book, *The Cognitive Structure of Scientific Revolutions* by Hanne Andersen, Peter Barker and Xiang Chen, published in Spring 2006 by Cambridge University Press. In the present paper I have not explained or defended the representation of concepts by frames, or the appearance in frames of separate lists for attributes and values. All I will say here is that there is extensive empirical evidence for all these claims and that the evidence is fully presented in the book. I have also significantly abbreviated the discussion of the Copernican revolution by not considering the main sixteenth century rival to Ptolemaic astronomy — the strictly earth-centered astronomy of the Averroists. Again, this story is told much more fully in the book. Galileo's telescopic discoveries assume a new importance against this background. In addition, I lack the space to give a detailed explanation of the transition from object concepts, with fixed values of their attributes, to event concepts with values that vary over time. In the book we explain how the introduction by Kepler of the event concept *orbit* marks a decisive change in the conceptual structure of astronomy away from the concepts I have discussed in this paper. In this cognitive reconstruction, the cognitive structure of the Copernican revolution begins with the concept of uniform circular motion, and ends with Kepler's introduction of the concept of an orbit.<sup>12</sup>

---

<sup>12</sup> I would like to thank Hanne Andersen, Xiang Chen, Kathleen Crowther, Katherine Tredwell, and Sylwester Ratowt for assistance with various drafts of the paper, and give my special thanks to Sylwester Ratowt for presenting the paper at the Cracow meeting in my place. He and the other people who have given me advice are not responsible for errors or infelicities.