

Roman Duda *

The Lvov School of mathematics

IN THE ANCIENT POLISH CITY OF LWÓW (in other languages Leopoldis, Lvov, Lemberg, Lviv) there was an old university, founded 1661 by the king Jan Kazimierz (1609–1672). During the period 1772–1918, when Poland has been partitioned by Russia, Prussia and Austria, the city came under Austrian domination and the university was temporarily closed. Reopened, with the German language of instruction (previously it was Latin), it did not enjoy high reputation. Only after Lvov became the capital of Galicia and the province was granted autonomy in 1860s (within Austro-Hungarian empire), the significance of the university has begun to raise. In 1871 Polish has been admitted as the language of instruction and soon some research has also started. In those times there were two chairs of mathematics at the university and two more at the Academy of Technology (Lvov Polytechnic since 1918). In 1910 university chairs of mathematics were occupied by Józef Puzyna (1856–1919) and Waclaw Sierpiński (1882–1969), both already known by their achievements. They became leaders of a group of talented youngsters, including Zygmunt Janiszewski (1888–1920), Stefan Mazurkiewicz (1888–1945), and Stanisław Ruziewicz (1889–1941), who later made their name in mathematics, and some others. Members of the group received results of some value, subsequently published in Polish, French and German journals, but the group itself has dispersed in 1914 after the outbreak of World War I: Puzyna became ill and died soon, Sierpiński was interned in Russia, Janiszewski volunteered for Polish military troops and died young, Mazurkiewicz moved to Warsaw; only Ruziewicz has remained in Lvov. However, around 1920, there appeared a new group of young mathematicians, this time with Hugo Steinhaus (1887–1972) and Stefan Banach (1892–1945) as new leaders, which flourished during the next two decades and became world-wide known as the Lvov School of Mathematics, a part of the interwar Polish School of Mathematics. Its most active members, besides the leaders, were Herman Auerbach (1901–1942), Meier Eidelheit (1910–1943), Mark Kac (1914–1984), Stefan Kaczmarz (1895–1939), Kazimierz Kuratowski (1896–1980, in the years 1927–1933 in Lvov), Antoni Łomnicki (1881–1941), Stanisław Mazur (1905–1981), Władysław Orlicz (1903–1990), Stanisław Ruziewicz (1889–1941), Juliusz Schauder (1899–1943), Stanisław Ulam (1909–1984). Sierpiński, Mazurkiewicz and other Warsaw mathematicians closely collaborated.

Main achievements of the Lvov School were in the area of *functional analysis* (Banach spaces, Hahn-Banach theorem on extending functionals, Banach-Steinhaus theorem on condensation of singularities, origins of duality theory, Orlicz spaces, some glimpses into nonlinear case, introduction of topological methods), *measure theory* (Banach-Tarski paradox, measurable cardinals) and *probability theory* (probability as a measure, independent functions), but important contributions were also made to the *theory of functions* (ineffective proofs of existence), *game theory* (game as a mathematical problem, strategy, value of a game, minimax), *algebra* (normed rings, Banach algebras), *partial differential equations* (Schauder-Leray index, elliptic and hyperbolic type), *recursive functions* etc. A characteristic feature of the Lvov School was frequent use of ineffective methods (Axiom of Choice, Baire category method, Lebesgue measure) in proving existential theorems. The development has been supported by the journal *Studia Mathematica* (founded 1929, 9 volumes until 1940, restarted 1948 in Wrocław) and confirmed by two monographs: *Théorie des opérations linéaires* by Banach (1932) and *Theorie der Orthogonalreihen* by Kaczmarz and Steinhaus (1936) as well as by Banach's plenary lecture at the Oslo Congress (1936). A characteristic feature of the Lvov mathematical life was its social flavor witnessed by frequent sessions of the Lvov branch of the Polish Mathematical Society (180 sessions with 360 communications in ten years 1928–1938), vivid contacts within country and abroad, and daily visits to the Scottisch Café, the place of famous *Scottish Book* in which nearly 200

* University of Wrocław, Wrocław, Poland; email: romanduda@poczta.onet.pl.

open problems were noted together with some comments and/or solutions. The Lvov School of Mathematics has soon gained high and friendly respect. It attracted many visitors from the country and from abroad, including almost all active Polish mathematicians and such figures as Ernest Zermelo (1871–1953), John von Neumann (1903–1957), Henri Lebesgue (1875–1941) and some others. *Studia Mathematica* were the first journal devoted specifically to functional analysis, to the development of which it highly contributed, since that time retaining high status of a leading journal. After 1945 a hand-written copy of the *Scottish Book* has been smuggled to the United States, translated there by Ulam and several times reedited. And most important, the School has strongly influenced development of mathematics in several fundamental directions.

Prospects for the future seemed rather good but the incredible catastrophe of World War II brought the School to an end. From about two dozens of its active members around half (12 names) perished during Soviet or Nazi occupation (shot dead, poisoned, emaciated to death), a few (including Kac and Ulam) managed to emigrate before the war, some (including Banach) died. And after 1945 all survived Poles in the area, including the living rest of mathematicians, were expelled from their home city because Allied Powers have decided that the eastern part of Poland (invaded by Stalin in 1939) was to become a part of the Soviet Union. In that way the Lvov School of Mathematics has been eventually eradicated and ceased to exist.